Robust Adaptive Neural Control for a Class of Perturbed Strict Feedback Nonlinear Systems

Shuzhi Sam Ge, *Senior Member, IEEE,* and Jing Wang, *Member, IEEE*

***Abstract*—This paper presents a robust adaptive neural control design for a class of perturbed strict feedback nonlinear system with both completely unknown virtual control coefficients and unknown nonlinearities. The unknown nonlinearities comprise two types of nonlinear functions: one naturally satisfies the "triangularity condition" and can be approximated by linearly parameterized neural networks, while the other is assumed to be partially known and consists of parametric uncertainties and known "bounding functions." With the utilization of iterative Lyapunov design and neural networks, the proposed design procedure expands the class of nonlinear systems for which robust adaptive control approaches have been studied. The design method does not require *a priori* knowledge of the signs of the unknown virtual control coefficients. Leakage terms are incorporated into the adaptive laws to prevent parameter drifts due to the inherent neural-network approximation errors. It is proved that the proposed robust adaptive scheme can guarantee the uniform ultimate boundedness of the closed-loop system signals. The control performance can be guaranteed by an appropriate choice of the design parameters. Simulation studies are included to illustrate the effectiveness of the proposed approach.**

***Index Terms*—Backstepping, neural networks, robust adaptive control, uncertain nonlinear systems.**

I. INTRODUCTION

ADAPTIVE control of nonlinear systems with parametric uncertainty has received a great deal of attention in the nonlinear control community [5], [11], [13]. Under the restrictions in the growth rate of nonlinearities and matching conditions [9], [21], adaptive control algorithms were first developed for linearizable nonlinear systems with unknown parameters. These restrictions were subsequently relaxed by the introduction of integrator backstepping design in [10], [13], and [22].

With the advances in adaptive nonlinear control, the more application-motivated problem of robust adaptive control for nonlinear systems in the presence of time-varying disturbances and unmodeled dynamics has gradually gained much attention. In an effort to enlarge the class of nonlinear uncertain systems for which adaptive backstepping control can be designed, recently a series of works have been focused on robust adaptive control of a class of nonlinear systems whose uncertainties include nonlinearly appearing parametric uncertainty, uncertain nonlinearities as well as unmeasured input-to-state stable dynamics [6], [7], [17], [25]. A robust adaptive nonlinear control design procedure was presented in [17] for a class of nonlinear systems with both parametric uncertainty and unknown nonlinearities under the assumption that unknown functions satisfy a so-called *triangular bounds* condition. The results extend the class of uncertain systems for which global adaptive stabilization methods can be applied. In [6] and [7], the authors proposed a robust adaptive control scheme for perturbed strict feedback nonlinear systems subject to nonlinear parametric uncertainty, uncertain nonlinearity, and unmodeled dynamics. The proposed robust adaptive controls in [6], [7], and [17] can guarantee the uniform ultimate boundedness of the closed-loop system signals. For a similar class of nonlinear system, [25] also presented an adaptive robust control method by combining the backstepping adaptive control with conventional deterministic robust control. The common features of the nonlinear systems discussed in [6], [7], [17], and [25] are that the system uncertainties are in the linearly parameterized forms and the system virtual control coefficients are assumed to be one.

In order to cope with highly uncertain nonlinear systems, as an alternative, approximator-based adaptive control approaches have also been extensively studied in the past decade using Lyapunov stability theory [2], [3], [18], [19], [23], [24], [27]. In [3] and [27], stable adaptive NN controllers were proposed for nonlinear systems in a Brunovsky form. The same system was studied in [23] and [24] by using fuzzy systems as function approximator and different adaptive fuzzy controllers have been derived. Using the idea of adaptive backstepping, the developed approximator-based adaptive control approaches were recently extended to nonlinear systems without satisfying matching condition [2], [18], [19]. In [2], by using a novel integral Lyapunov function, an adaptive backstepping controller was presented for nonlinear strict-feedback systems. The possible controller singularity problem is avoided without using projection. In [19], a stable adaptive neural control method was presented for a second-order nonlinear system, where the unknown system function was parameterized by radial basis function (RBF) neural networks, and unknown neural reconstruction error bound was also adaptively tuned online. By using backstepping and online approximators, the result in [19] was extended to a class of strict feedback nonlinear systems with unknown control coefficients in [18], where the signs of control coefficients are assumed to be known.

In this paper, we present a robust adaptive neural control design procedure for a class of single-input–single-output

Manuscript received October 24, 2001; revised June 24, 2002.

S. S. Ge is with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576, Singapore (e-mail: elegesz@nus.edu.sg).

J. Wang was with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576, Singapore. He is now with the School of Electrical Engineering and Computer Science, University of Central Florida, Orlando, FL 32816 USA.

Digital Object Identifier 10.1109/TNN.2002.804306

(SISO) nonlinear uncertain systems in the perturbed strict feedback form

$$\begin{aligned}\dot{x}_i &= g_i x_{i+1} + f_i(\overline{x}_i) + \Delta_i(t, x), \qquad i = 1, \ldots, n-1\\ \dot{x}_n &= g_n \beta(x) u + f_n(x) + \Delta_n(t, x)\\ y &= x_1\end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ is the state vector, $\overline{x}_i = [x_1, \ldots, x_i]^T$, $u \in R$ is the control, $f_1, \ldots, f_n$ are unknown smooth nonlinear functions, $\beta(x)$: $R^n \rightarrow R$ is a known smooth function and $\beta(x) \neq 0, \forall x \in R^n$, g_i, $i = 1, \ldots, n$ are unknown constants, and they are referred to as virtual control coefficients, in particular, g_n is referred to as the high-frequency gain, and Δ_is are unknown Lipschitz continuous functions.

It is noticed that robust adaptive control algorithms for system (1) have been developed in [6], [7], [17], and [19], when virtual control coefficients $g_i = 1$. The problem of adaptive control of systems with unknown virtual control coefficients has also received much attention in recent years. In [13], under the assumption of unknown g_is but with known signs of g_is, an adaptive control solution was presented for strict feedback nonlinear systems without disturbance term Δ_is. When there is no *a priori* knowledge about the signs of virtual control coefficients, the problem becomes much more difficult. The first solution was given in [16] for a class of first-order linear systems, where the Nussbaum-type gain was originally proposed. This method was then generalized to higher order linear systems in [15]. For nonlinear systems, some results have also been reported in the literature. Using Nussbaum gain, an adaptive control algorithm was first given in [14] for first-order nonlinear systems. In [8], a nonlinear robust control scheme was proposed, which can identify the unknown control directions and guarantee global stability of closed-loop system. The design procedure proposed in [8] can only be applied to second-order systems, and bounds on the uncertainties and their partial derivatives need to be known. Recently, for a class of high-order nonlinear systems in the parameter strict feedback form, adaptive control designs with unknown signs of virtual control coefficients have been developed in [1] and [26] by using Nussbaum gain. The discussed systems are focused on the so-called strict feedback nonlinear systems under the assumptions that the system uncertainties have been linearly parameterized and there are no disturbance terms within the systems. To the best of the authors' knowledge, few results are available for the robust adaptive control of perturbed strict feedback nonlinear systems with unknown virtual control coefficients in the literature. The standard adaptive NN control approaches cannot be extended to the control of the perturbed strict feedback system (1) to cope with the disturbance terms Δ_is due to the key assumption of known signs of virtual control coefficients is employed in these works.

In this paper, by using neural networks to approximate the unknown nonlinear functions $f_i(\cdot)$ in (1), a robust adaptive neural controller is proposed based on iterative Lyapunov design. The design method do not require the *a priori* knowledge of the signs of the unknown virtual control coefficients due to the incorporation of Nussbaum gain in the controller design. The unknown bounds of both neural-network approximation errors and disturbance terms are estimated online. Leakage terms are incorporated into the adaptive laws to prevent the parameters drift due to the inherent neural-network approximation errors. The proposed design method expands the class of nonlinear systems for which robust adaptive control approaches have been studied. It has been proven that the proposed robust adaptive scheme can guarantee the uniform ultimate boundedness of the closed-loop system signals. The control performance can be guaranteed by appropriately choosing the design parameters.

This paper is organized as follows. Section II presents some assumptions and the structure of the linearly parameterized neural networks used in controller design. Section III proposes the robust adaptive control design procedure for perturbed strict feedback nonlinear system with completely unknown virtual control coefficients and gives the main result of the paper. Section IV contains a simulation example to show the effectiveness of the proposed controller.

II. Preliminaries

A. Problem Statements

Consider the control problem of a single-input–single-output (SISO) nonlinear uncertain system transformable into (1). The control objective is to construct a robust adaptive nonlinear control law so that the output y of the above system is driven to a small neighborhood of the origin, while keeping internal Lagrange stability.

The system described by (1) is in the so-called semistrict feedback form [25], which has two types of unknown nonlinear functions: one naturally satisfies the "triangularity condition" and can be directly approximated by linearly parameterized approximators; while the other, arises owing to $\Delta_i(t, x)$, is assumed to be partially known and consists of parametric uncertainties and known "bounding functions." The unknown nonlinear functions $\Delta_i(t, x)$ could be due to many factors [17], such as measurement noise, modeling errors, external disturbances, modeling simplifications or changes due to time variations.

Remark 2.1: Under the assumptions that the virtual control coefficients $g_i = 1$ and the unknown function $f_i(\overline{x}_i)$s are linearly parameterized as $\theta_i^T \psi_i(\overline{x}_i)$ with θ_i being the unknown constant parameters vector, several adaptive robust control algorithms for semistrict feedback nonlinear systems (1) have been developed in [6], [7], [17], and [25]. However, for more general class of nonlinear uncertain systems like (1), few results are available in the literature. In this paper, we relax these two assumptions and propose a robust adaptive backstepping design method for system (1) by combining neural-network approximators and backstepping together. The desired adaptive nonlinear controller is explicitly designed via a recursive robust adaptive backstepping algorithm.

Throughout the paper, the following assumption will be imposed on system (1).

Assumption 2.1: For $1 \leq i \leq n$, there exists unknown positive constant p_i^* such that $\forall (t, x) \in R_+ \times R^n$

$$|\Delta_i(t, x)| \leq p_i^* \phi_i(\overline{x}_i) \tag{2}$$

where ϕ_i is a known nonnegative smooth function.

Remark 2.2: Assumption 2.1 implies that the allowed class of uncertainties Δ_i satisfies a triangularity condition in terms of x. This will be exploited for the ease of our controller design. Similar assumptions to Assumption 2.1 have been used in [6], [7], [17], and [25]. In this paper, we do not need the exact expression of $\Delta_i(t, x) = \phi_i(\overline{x}_i)p_i$ as investigated in [13], where it showed that the existence of disturbance terms $\phi_i(\overline{x}_i)p_i$ might drive the system states escape to infinity in finite time, even if p_i is an exponentially decaying disturbance.

In order to cope with the unknown signs of virtual control coefficients g_i, the Nussbaum gain technique is employed in this paper. A function $N(\zeta)$ is called a Nussbaum-type function [16] if it has the following properties:

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta)\, d\zeta = \infty \tag{3}$$

$$\lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta)\, d\zeta = -\infty. \tag{4}$$

Commonly used Nussbaum functions include: $k^2\cos(k)$, $k^2\sin(k)$, and $\exp(k^2)\cos((\pi/2)k)$ [4]. In this paper, an even Nussbaum function $\exp(k^2)\cos((\pi/2)k)$ is exploited. Physically, they can be visualized as functions of infinite gains and infinite switching frequency.

The following lemma regarding to the property of Nussbaum gain is used in the controller design and theorem proof of next section.

Lemma 1: Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0$, $\forall\, t \in [0, t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \leq C_0 + e^{-C_1 t}\int_0^t \left(g_1 N(\zeta)\dot{\zeta} + \dot{\zeta}\right) e^{C_1\tau}\, d\tau \qquad \forall\, t \in [0, t_f) \tag{5}$$

where constant $C_1 > 0$, g_1 is a nonzero constant and C_0 represents some suitable constant, then $V(t)$, $\zeta(t)$ and $\int_0^t g_1 N(\zeta)\dot{\zeta}\, d\tau$ must be bounded on $[0, t_f)$.

Proof: See the Appendix.

Remark 2.3: Note that (5) can be converted into the following inequality:

$$0 < V(t) \leq C \pm |g_1|\int_0^{\zeta(t)} N(\zeta)\, d\zeta \pm \zeta(t)$$
$$\text{constant } C = C_0 \mp \zeta(0), \quad \forall\, t \in [0, t_f)$$

that does not explicitly depend on time as given in (91) in the Appendix. As the discussed system does not explicitly depend on time, and the bounded $V(t)$ and $\zeta(t)$ are not depending on time explicitly, uniform conclusion and $t_f = \infty$ can be made in the controller design later.

B. Linearly Parameterized Neural Networks

A linearly parameterized approximator shall be used to approximate the unknown nonlinearities $f_i(\cdot)$. Several function approximators can be applied for this purpose, e.g., RBF neural networks [2], [20], high-order neural networks [12], and fuzzy systems [24], which can be described as $W^T S(z)$ with input vector $z \in R^n$, weight vector $W \in R^l$, node number l, and basis function vector $S(z) \in R^l$. Universal approximation results indicate that, if l is chosen sufficiently large, then $W^T S(z)$ can approximate any continuous function to any desired accuracy over a compact set [12], [20]. In this paper, we use the following RBF NN to approximate a smooth function

$$h_{nn}(z) = W^T S(z) \tag{6}$$

where the input vector $z \in \Omega \subset R^n$, weight vector $W = [w_1, w_2, \ldots, w_l]^T \in R^l$, the NN node number $l > 1$, and $S(z) = [s_1(z), \ldots, s_l(z)]^T$, with $s_i(z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(z) = \exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\eta_i^2}\right], \qquad i = 1, 2, \ldots, l \tag{7}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

For the unknown nonlinear functions $f_i(\overline{x}_i)$, $i = 1, \ldots, n$ in (1), we have the following approximation over the compact sets Ω_i:

$$f_i(\overline{x}_i) = W_i^{*T}\psi_i(\overline{x}_i) + \omega_i(\overline{x}_i), \qquad \forall\, \overline{x}_i \in \Omega_i \subset R^i \tag{8}$$

where $\psi(\overline{x}_i)$ is the basis function vector, $\omega_i(\overline{x}_i)$ is the approximation error, and W_i^* is an unknown constant parameter vector.

Remark 2.4: The optimal weight vector W_i^* in (8) is an "artificial" quantity required only for analytical purposes. Typically, W_i^* is chosen as the value of W_i that minimizes $\omega_i(\overline{x}_i)$ for all $\overline{x}_i \in \Omega_i$, where $\Omega_i \subset R^i$ is a compact set, i.e.,

$$W_i^* := \arg\min_{W_i \in R^n}\left\{\sup_{\overline{x}_i \in \Omega_i}\left|f_i(\overline{x}_i) - W_i^T\psi(\overline{x}_i)\right|\right\}. \tag{9}$$

We make the following assumption on the approximation error.

Assumption 2.2: Over a compact region $\Omega_i \in R^i$

$$|\omega_i(\overline{x}_i)| \leq \delta_i^* \qquad \forall\, \overline{x}_i \in \Omega_i,\ i = 1, \ldots, n \tag{10}$$

where $\delta_i^* \geq 0$ is an unknown bound.

From the above analysis, we see that the system uncertainties are converted to the estimation of unknown parameters W_i^* and unknown augmented parameters consisting of p_i^*, δ_i^*, and g_i, as will be detailed later.

Remark 2.5: In practice, the occurrence of control coefficients g_i are quite common. The examples range from electric motors and robotic manipulators to flight dynamics. While it is common that systems have unknown g_i but with known sign, it is also quite possible that the uncertain system contains completely unknown control coefficients. In this paper, we present a robust adaptive control solution for a class of perturbed strict feedback nonlinear system with completely unknown control coefficients.

Remark 2.6: For other linear-in-the-parameters function approximators such as fuzzy systems, polynomials, splines, and wavelet networks, the controllers presented in this paper using RBF NN can be replaced by these function approximators without any difficulty.

III. Robust Adaptive Control Design

In this section, the robust adaptive control design procedure for nonlinear system (1) is presented.

Using (8), (1) can be expressed as

$$\begin{aligned} \dot{x}_i &= g_i x_{i+1} + W_i^{*T}\psi_i(\overline{x}_i) + \omega_i(\overline{x}_i) + \Delta_i(t, x), \\ &\quad i = 1, \ldots, n-1 \\ \dot{x}_n &= g_n \beta(x) u + W_n^{*T}\psi_n(x) + \omega_n(x) + \Delta_n(t, x) \\ y &= x_1. \end{aligned} \tag{11}$$

Remark 3.1: The results obtained in this paper are semiglobal, in the sense that they are valid as long as $\overline{x}_i(t)$ remains in Ω_i, where the set Ω_i and bounding parameter δ_i^* can be arbitrarily large. In the special case that (10) holds for all $\overline{x}_i \in R^i$, then the stability results become global.

The system described by (11) has three types of uncertainty: parametric uncertainty, which arises due to the unknown W_i^*, the bounding uncertainty that arises due to the unknown bounds on Δ_i and ω_i, and unknown virtual control coefficient g_i.

Our design consists of n steps. The design of both the control law and the adaptive laws is based on a change of coordinates

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 - \alpha_1(x_1, \hat{W}_{a,1}, \hat{b}_1) \\ &\vdots \\ z_i &= x_i - \alpha_{i-1}(x_1, \ldots, x_{i-1}, \hat{W}_{a,1}, \ldots, \\ &\qquad \hat{W}_{a,i-1}, \hat{b}_1, \ldots, \hat{b}_{i-1}) \\ &\vdots \\ z_n &= x_n - \alpha_{n-1}(x_1, \ldots, x_{n-1}, \hat{W}_{a,1}, \ldots \\ &\qquad \hat{W}_{a,n-1}, \hat{b}_1, \ldots, \hat{b}_{n-1}) \end{aligned} \tag{12}$$

where the functions α_i, $i = 1, \ldots, n-1$ are referred to as intermediate control functions which will be designed later, $\hat{b}_i$ is the parameter estimate for b_i^* which is the grouped unknown bound for p_i^* and δ_i^*, and $\hat{W}_{a,i}$ represents the estimate of unknown parameter vector $W_{a,i}^*$ which is an augmented parameter vector consisting of g_j, $j = 1, \ldots, i-1$ and W_j^*, $j = 1, \ldots, i$, as clarified later. At each intermediate step i, we design the intermediate control function α_i using an appropriate Lyapunov function V_i, and give the parameters update laws $\dot{\hat{b}}_i$ and $\dot{\hat{W}}_{a,i}$. At the nth step, the actual control u appears and the design is completed.

Step 1: To start, let us study the following subsystem of (11)

$$\dot{x}_1 = g_1 x_2 + W_1^{*T}\psi_1(x_1) + \omega_1(x_1) + \Delta_1(t, x) \tag{13}$$

where x_2 is taken as a virtual control input.

To design a stabilizing adaptive control law for system (13), consider a Lyapunov function candidate V_0

$$V_0(x_1) = \tfrac{1}{2} x_1^2. \tag{14}$$

In light of Assumptions 2.1 and 2.2, the time derivative of V_0 along the solution of (13) satisfies

$$\begin{aligned} \dot{V}_0 &= x_1\left(g_1 x_2 + W_1^{*T}\psi_1(x_1) + \omega_1 + \Delta_1(t, x)\right) \\ &\le x_1\left(g_1 x_2 + W_1^{*T}\psi_1(x_1)\right) + \delta_1^*|x_1| + p_1^*|x_1|\phi_1(x_1) \\ &\le x_1\left(g_1 x_2 + W_1^{*T}\psi_1(x_1)\right) + b_1^*|x_1|\overline{\phi}_1(x_1) \end{aligned} \tag{15}$$

where

$$b_1^* = \max\{\delta_1^*, p_1^*\} \tag{16}$$

$$\overline{\phi}_1(x_1) = 1 + \phi_1(x_1). \tag{17}$$

For notational convenience, let $W_{a,1}^* = W_1^*$, $\psi_{a,1} = \psi_1$. Consider the Lyapunov function candidate V_1

$$V_1 = V_0 + \tfrac{1}{2}(\hat{W}_{a,1} - W_{a,1}^*)^T\Gamma_1^{-1}(\hat{W}_{a,1} - W_{a,1}^*) + \frac{1}{2\lambda_1}(\hat{b}_1 - b_1^*)^2 \tag{18}$$

where $\Gamma_1 = \Gamma_1^T > 0$, $\lambda_1 > 0$, and $\hat{W}_{a,1}$ and $\hat{b}_1$ are the parameters estimates to be determined later.

Then, the time derivative of V_1 along (15) is

$$\begin{aligned} \dot{V}_1 &\le x_1\left(g_1 x_2 + W_{a,1}^{*T}\psi_{a,1}(x_1)\right) + b_1^*|x_1|\overline{\phi}_1(x_1) \\ &\quad + (\hat{W}_{a,1} - W_{a,1}^*)^T\Gamma_1^{-1}\dot{\hat{W}}_{a,1} + \frac{1}{\lambda_1}(\hat{b}_1 - b_1^*)\dot{\hat{b}}_1. \end{aligned} \tag{19}$$

Let the intermediate control function α_1 be

$$\alpha_1(x_1, \hat{W}_{a,1}, \hat{b}_1) = N(\zeta_1)\left(z_1 + \hat{W}_{a,1}^T\psi_{a,1}(x_1) + \hat{b}_1\overline{\phi}_1(x_1) \cdot \tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right]\right) \tag{20}$$

with

$$N(\zeta_1) = \exp(\zeta_1^2)\cos\left(\frac{\pi}{2}\zeta_1\right) \tag{21}$$

$$\dot{\zeta}_1 = z_1^2 + z_1\hat{W}_{a,1}\psi_{a,1} + z_1\hat{b}_1\overline{\phi}_1 \tanh\left[\frac{z_1\overline{\phi}_1}{\epsilon_1}\right] \tag{22}$$

where ϵ_1 is a small positive constant and $N(\zeta_1)$ is an even smooth Nussbaum-type function.

Remark 3.2: Note that for system (11), if there are no uncertain terms ω_i and Δ_i, then by regrouping parameters $W_1^*, \ldots, W_n^*$ as $\theta^* = [W_1^{*T}, \ldots, W_n^{*T}]^T$ and letting $\phi_i = [0_1^T, \ldots, 0_{i-1}^T, \psi_i^T, 0_{i+1}^T, \ldots, 0_n^T]^T$, where $0_j := [0, \ldots, 0]^T \in R^{l_j}$, l_j is the node number of neural network $W_j^{*T}\psi_j$, the system (11) becomes

$$\begin{aligned} \dot{x}_i &= g_i x_{i+1} + \theta^{*T}\phi_i, \qquad 1 \le i \le n-1 \\ \dot{x}_n &= g_n u + \theta^{*T}\phi_n \end{aligned} \tag{23}$$

which is in the strict feedback form as discussed in [1] and [26], where the uncertainty in the system is assumed to be due to unknown parameters only and unknown parameters are appearing linearly with respect to known nonlinear functions. However, when involving the neural-network approximation errors and uncertainty terms Δ_i, the problem becomes much more difficult. Different from the work in [26], in this paper, with the aid of neural networks, we present a robust adaptive neural control scheme to deal with both unknown parameters and unknown uncertainty functions Δ_i due to modeling errors, external disturbances or a combination of these.

As in [17], in order to prevent parameter drifts, we present the following adaptive law incorporating a leakage term based on a variation of σ-modification. Let the parameter adaptation laws be

$$\dot{\hat{W}}_{a,1} = \Gamma_1 x_1 \psi_{a,1}(x_1) - \Gamma_1\sigma_{W_1}(\hat{W}_{a,1} - W_{a,1}^0) \tag{24}$$

$$\dot{\hat{b}}_1 = \lambda_1 x_1 \overline{\phi}_1(x_1)\tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right] - \lambda_1\sigma_{b_1}(\hat{b}_1 - b_1^0) \tag{25}$$

where $\sigma_{W_1} > 0, \sigma_{b_1} > 0$ and $W_{a,1}^0, b_1^0 > 0$ are design constants.

Using (20), a direct substitution of $x_2 = z_2 + \alpha_1$ into (19) gives

$$\begin{aligned}\dot{V}_1 \le\, & g_1 z_1 z_2 + g_1 N(\zeta_1)\left(z_1^2 + z_1 \hat{W}_{a,1}^T \psi_{a,1}(x_1)\right.\\ &\left. + z_1 \hat{b}_1 \overline{\phi}_1(x_1)\tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right]\right)\\ &+ z_1 W_{a,1}^{*T}\psi_{a,1}(x_1) + b_1^*|x_1|\overline{\phi}_1(x_1)\\ &+ (\hat{W}_{a,1} - W_{a,1}^*)^T\Gamma_1^{-1}\dot{\hat{W}}_{a,1} + \frac{1}{\lambda_1}(\hat{b}_1 - b_1^*)\dot{\hat{b}}_1. \quad (26)\end{aligned}$$

Adding and subtracting

$$z_1^2 + z_1 \hat{W}_{a,1}^T\psi_{a,1}(x_1) + z_1\hat{b}_1\overline{\phi}_1(x_1)\tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right]$$

on the right hand of (26), and noting (22), (24), and (25), we have

$$\begin{aligned}\dot{V}_1 \le & -z_1^2 + g_1 z_1 z_2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1\\ &+ b_1^*|x_1|\overline{\phi}_1(x_1) - b_1^* x_1\overline{\phi}_1(x_1)\tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right]\\ &- \sigma_{W_1}(\hat{W}_{a,1} - W_{a,1}^*)^T(\hat{W}_{a,1} - W_{a,1}^0)\\ &- \sigma_{b_1}(\hat{b}_1 - b_1^*)(\hat{b}_1 - b_1^0). \quad (27)\end{aligned}$$

By completing the squares

$$\begin{aligned}&\sigma_{W_1}(\hat{W}_{a,1} - W_{a,1}^*)^T(\hat{W}_{a,1} - W_{a,1}^0)\\ &= \tfrac{1}{2}\sigma_{W_1}\left\|\hat{W}_{a,1} - W_{a,1}^*\right\|^2 + \tfrac{1}{2}\sigma_{W_1}\left\|\hat{W}_{a,1} - W_{a,1}^0\right\|^2\\ &\quad - \tfrac{1}{2}\sigma_{W_1}\left\|W_{a,1}^* - W_{a,1}^0\right\|^2 \quad (28)\end{aligned}$$

$$\begin{aligned}&\sigma_{b_1}(\hat{b}_1 - b_1^*)(\hat{b}_1 - b_1^0)\\ &= \tfrac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^*)^2 + \tfrac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^0)^2 - \tfrac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2 \quad (29)\end{aligned}$$

and using the following nice property with regard to function $\tanh(\cdot)$ [17]:

$$0 \le |x| - x\tanh\left(\frac{x}{\epsilon}\right) \le 0.2785\epsilon, \qquad \text{for } \epsilon > 0,\ x \in R. \quad (30)$$

Equation (27) can be further written as

$$\begin{aligned}\dot{V}_1 \le & -z_1^2 + g_1 z_1 z_2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1\\ &- \tfrac{1}{2}\sigma_{W_1}\left\|\hat{W}_{a,1} - W_{a,1}^*\right\|^2 - \tfrac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^*)^2\\ &+ b_1^* 0.2785\epsilon_1 + \tfrac{1}{2}\sigma_{W_1}\left\|W_{a,1}^* - W_{a,1}^0\right\|^2\\ &+ \tfrac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2\\ \le & -\tfrac{3}{4}z_1^2 - \left(\tfrac{1}{4}z_1 - g_1 z_2\right)^2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1\\ &- \tfrac{1}{2}\sigma_{W_1}\left\|\hat{W}_{a,1} - W_{a,1}^*\right\|^2\\ &- \tfrac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^*)^2 + b_1^* 0.2785\epsilon_1\\ &+ \tfrac{1}{2}\sigma_{W_1}\left\|W_{a,1}^* - W_{a,1}^0\right\|^2 + \tfrac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2 + g_1^2 z_2^2\\ \le & -\tfrac{3}{4}z_1^2 - \tfrac{1}{2}\sigma_{W_1}\left\|\hat{W}_{a,1} - W_{a,1}^*\right\|^2 - \tfrac{1}{2}\sigma_{b_1}(\hat{b}_1 - b_1^*)^2\\ &+ g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + b_1^* 0.2785\epsilon_1\\ &+ \tfrac{1}{2}\sigma_{W_1}\left\|W_{a,1}^* - W_{a,1}^0\right\|^2 + \tfrac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2 + g_1^2 z_2^2. \quad (31)\end{aligned}$$

This yields

$$\dot{V}_1 \le -C_{11}V_1 + C_{12} + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + g_1^2 z_2^2 \quad (32)$$

where the constants $C_{11} > 0$ and $C_{12} > 0$ are defined as

$$C_{11} := \min\left\{\frac{3}{2}, \frac{\sigma_{W_1}}{\lambda_{\max}(\Gamma_1^{-1})}, \sigma_{b_1}\lambda_1\right\} \quad (33)$$

$$\begin{aligned}C_{12} := & b_1^* 0.2785\epsilon_1 + \tfrac{1}{2}\sigma_{W_1}\left\|W_{a,1}^* - W_{a,1}^0\right\|^2\\ &+ \tfrac{1}{2}\sigma_{b_1}(b_1^* - b_1^0)^2. \quad (34)\end{aligned}$$

Let $\rho_1 := C_{12}/C_{11}$, upon multiplication of (32) by $e^{C_{11}t}$, it becomes

$$\begin{aligned}\frac{d}{dt}\left(V_1(t)e^{C_{11}t}\right) \le & C_{12}e^{C_{11}t} + g_1 N(\zeta_1)\dot{\zeta}_1 e^{C_{11}t}\\ &+ \dot{\zeta}_1 e^{C_{11}t} + g_1^2 z_2^2 e^{C_{11}t}. \quad (35)\end{aligned}$$

Integrating (35) over $[0, t]$, we have

$$\begin{aligned}0 &\le V_1(t)\\ &\le \rho_1 + [V_1(0) - \rho_1]e^{-C_{11}t}\\ &\quad + e^{-C_{11}t}\int_0^t (g_1 N(\zeta_1) + 1)\, e^{C_{11}\tau}\dot{\zeta}_1\, d\tau\\ &\quad + e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau\\ &\le \rho_1 + V_1(0) + e^{-C_{11}t}\int_0^t g_1 N(\zeta_1)\dot{\zeta}_1 e^{C_{11}\tau}\, d\tau\\ &\quad + e^{-C_{11}t}\int_0^t \dot{\zeta}_1 e^{C_{11}\tau}\, d\tau + e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau. \quad (36)\end{aligned}$$

Remark 3.3: Noting (36), if there is no extra term $e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau$ within the inequality, we can conclude from Lemma 1 that $V_1(t)$, ζ_1 and $z_1, \hat{W}_{a,1}, \hat{b}_1$ are all bounded on $[0, t_f)$. Thus, no finite-time escape phenomenon may happen and $t_f = \infty$, and we claim that $z_1, \hat{W}_{a,1}, \hat{b}_1$ are uniformly ultimately bounded. Due to the undesired term $e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau$ in (36), Lemma 1 cannot be directly used. However, because

$$\begin{aligned}e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau &\le e^{-C_{11}t} g_1^2 \sup_{\tau\in[0,t]} z_2^2 \int_0^t e^{C_{11}\tau}\, d\tau\\ &\le \frac{g_1^2 \sup_{\tau\in[0,t]} z_2^2}{C_{11}} \quad (37)\end{aligned}$$

thus, if z_2 can be regulated as bounded, from (37), the boundedness of $e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau$ is obvious. Then, according to Lemma 1, the boundedness of $z_1(t)$ can be guaranteed. Therefore, the effect of $e^{-C_{11}t}\int_0^t g_1^2 z_2^2 e^{C_{11}\tau}\, d\tau$ will be dealt with in the following steps.

Step i $(2 \le i \le n-1)$: A similar procedure is employed recursively for each step $i = 2, \ldots, n-1$. The derivative of $(1/2)z_i^2$ is

$$\begin{aligned}z_i\dot{z}_i = z_i\Bigg[& g_i x_{i+1} + W_i^{*T}\psi_i + \omega_i + \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\\ &\cdot (g_j x_{j+1} + W_j^{*T}\psi_j + \omega_j + \Delta_j)\\ &- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_{a,j}}\dot{\hat{W}}_{a,j} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j\Bigg]. \quad (38)\end{aligned}$$

In view of Assumptions 2.1 and 2.2, we have

$$
\begin{aligned}
z_i\left(\Delta_i(t,x) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j(t,x) + \omega_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\omega_j\right)
&\le |z_i|\left[p_i^*\phi_i + \sum_{j=1}^{i-1}p_j^*\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\phi_j + \delta_i^* + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\delta_j^*\right]\\
&\le b_i^*|z_i|\overline{\phi}_i(\overline{x}_i)
\end{aligned}\tag{39}
$$

where $b_i^* = \max\{p_1^*, \ldots, p_i^*, \delta_1^*, \ldots, \delta_i^*\}$, and

$$
\overline{\phi}_i(\overline{x}_i) \ge \phi_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\phi_j + 1 + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|
$$

is a smooth positive function.

It is easy to find a smooth positive function

$$
\overline{\phi}_i \ge \phi_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\phi_j + 1 + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|. \tag{40}
$$

A simple example is

$$
\overline{\phi}_i = \phi_i + 1 + \sum_{j=1}^{i-1}\left(\frac{1}{4}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 + 1\right)(\phi_j + 1). \tag{41}
$$

Thus, (38) can be rewritten as

$$
z_i\dot{z}_i \le z_i\left[g_i x_{i+1} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_j x_{j+1} + W_i^{*T}\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}W_j^{*T}\psi_j + \beta_i\right] + b_i^*|z_i|\overline{\phi}_i \tag{42}
$$

where

$$
\beta_i = -\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{W}_{a,j}}\dot{\hat{W}}_{a,j} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j.
$$

Let

$$
W_{a,i}^* = [1, g_1, \ldots, g_{i-1}, W_i^{*T}, W_1^{*T}, \ldots, W_{i-1}^{*T}]^T \tag{43}
$$

$$
\psi_{a,i} = \left[\beta_i, -\frac{\partial\alpha_{i-1}}{\partial x_1}x_2, \ldots, -\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}x_i, \psi_i^T, -\frac{\partial\alpha_{i-1}}{\partial x_1}\psi_1^T, \ldots, -\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\psi_{i-1}^T\right]^T \tag{44}
$$

then, (42) can be further written as

$$
z_i\dot{z}_i \le z_i\left[g_i x_{i+1} + W_{a,i}^{*T}\psi_{a,i}\right] + b_i^*|z_i|\overline{\phi}_i. \tag{45}
$$

Consider the Lyapunov function candidate

$$
V_i = \tfrac{1}{2}z_i^2 + \tfrac{1}{2}(\hat{W}_{a,i} - W_{a,i}^*)^T\Gamma_i^{-1}(\hat{W}_{a,i} - W_{a,i}^*) + \frac{1}{2\lambda_i}(\hat{b}_i - b_i^*)^2 \tag{46}
$$

where $\Gamma_i = \Gamma_i^T > 0$, $\lambda_i > 0$, and $\hat{W}_{a,i}$ and $\hat{b}_i$ are the parameters estimates to be determined later. Differentiating V_i with respect to time gives

$$
\dot{V}_i \le z_i\left[g_i x_{i+1} + W_{a,i}^{*T}\psi_{a,i}\right] + b_i^*|z_i|\overline{\phi}_i + (\hat{W}_{a,i} - W_{a,i}^*)^T\Gamma_i^{-1}\dot{\hat{W}}_{a,i} + \frac{1}{\lambda_2}(\hat{b}_i - b_i^*)\dot{\hat{b}}_i. \tag{47}
$$

By selecting α_i and parameters adaptation laws as

$$
\alpha_i = N(\zeta_i)\left(z_i + \hat{W}_{a,i}^T\psi_{a,i} + \hat{b}_i\overline{\phi}_i\tanh\left[\frac{z_i\overline{\phi}_i}{\epsilon_i}\right]\right) \tag{48}
$$

$$
N(\zeta_i) = \exp(\zeta_i^2)\cos\left(\frac{\pi}{2}\zeta_i\right) \tag{49}
$$

$$
\dot{\zeta}_i = z_i^2 + z_i\hat{W}_{a,i}\psi_{a,i} + z_i\hat{b}_i\overline{\phi}_i\tanh\left[\frac{z_i\overline{\phi}_i}{\epsilon_i}\right] \tag{50}
$$

$$
\dot{\hat{W}}_{a,i} = \Gamma_i z_i\psi_{a,i} - \Gamma_i\sigma_{W_i}\left(\hat{W}_{a,i} - W_{a,i}^0\right) \tag{51}
$$

$$
\dot{\hat{b}}_i = \lambda_i z_i\overline{\phi}_i\tanh\left[\frac{z_i\overline{\phi}_i}{\epsilon_i}\right] - \lambda_i\sigma_{b_i}(\hat{b}_i - b_i^0) \tag{52}
$$

where ϵ_i is a small positive constant and $\sigma_{W_i} > 0$, $\sigma_{b_i} > 0$ and $W_{a,i}^0$, $b_i^0 > 0$ are design constants.

Using the same techniques as done previously, we obtain

$$
\begin{aligned}
\dot{V}_i \le& -z_i^2 + g_i z_i z_{i+1} + g_i N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i
- \tfrac{1}{2}\sigma_{W_i}\left|\hat{W}_{a,i} - W_{a,i}^*\right|^2 - \tfrac{1}{2}\sigma_{b_i}(\hat{b}_i - b_i^*)^2 + b_i^* 0.2785\epsilon_i\\
&+ \tfrac{1}{2}\sigma_{W_{a,i}}\left|W_{a,i}^* - W_{a,i}^0\right|^2 + \tfrac{1}{2}\sigma_{b_i}(b_i^* - b_i^0)^2\\
\le& -\tfrac{3}{4}z_i^2 + g_i N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - \tfrac{1}{2}\sigma_{W_i}\left|\hat{W}_{a,i} - W_{a,i}^*\right|^2
- \tfrac{1}{2}\sigma_{b_i}(\hat{b}_i - b_i^*)^2 + b_i^* 0.2785\epsilon_i + \tfrac{1}{2}\sigma_{W_i}\left|W_{a,i}^* - W_{a,i}^0\right|^2\\
&+ \tfrac{1}{2}\sigma_{b_i}(b_i^* - b_i^0)^2 + g_i^2 z_{i+1}^2.
\end{aligned}\tag{53}
$$

Similarly, this yields

$$
\begin{aligned}
0 \le V_i(t) &\le \rho_i + V_i(0) + e^{-C_{i1}t}\int_0^t g_i N(\zeta_i)\dot{\zeta}_i e^{C_{i1}\tau}\,d\tau\\
&+ e^{-C_{i1}t}\int_0^t \dot{\zeta}_i e^{C_{i1}\tau}\,d\tau + e^{-C_{i1}t}\int_0^t g_i^2 z_{i+1}^2 e^{C_{i1}\tau}\,d\tau
\end{aligned}\tag{54}
$$

where $\rho_i := C_{i2}/C_{i1}$, the constants $C_{i1} > 0$ and $C_{i2} > 0$ are defined as

$$
C_{i1} := \min\left\{\frac{3}{2}, \frac{\sigma_{W_i}}{\lambda_{\max}(\Gamma_i^{-1})}, \sigma_{b_i}\lambda_i\right\} \tag{55}
$$

$$
C_{i2} := b_i^* 0.2785\epsilon_i + \tfrac{1}{2}\sigma_{W_i}\left\|W_{a,i}^* - W_{a,i}^0\right\|^2 + \tfrac{1}{2}\sigma_{b_i}(b_i^* - b_i^0)^2. \tag{56}
$$

Remark 3.4: Similar to the discussion in Remark 3.3, if z_{i+1} can be regulated as bounded such that $e^{-C_{i1}t}\int_0^t g_i^2 z_{i+1}^2 e^{C_{i1}\tau}\,d\tau$ is bounded at the following steps, then, according to Lemma 1, the boundedness of $z_i(t)$ and $z_i(t) \in L_2$ can be guaranteed.

Step n: In this final step, the actual control u appears. Similarly, we have

$$z_n\dot{z}_n \le z_n\left[g_n\beta(x)u + W_n^{*T}\psi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\cdot(g_jx_{j+1}+W_j^{*T}\psi_j) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{W}_{a,j}}\dot{\hat{W}}_{a,j} - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j\right] + b_n^*|z_n|\overline{\phi}_n$$
$$= z_n\left[g_n\beta(x)u + W_n^{*T}\psi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\cdot(g_jx_{j+1}+W_j^{*T}\psi_j) + \beta_n\right] + b_n^*|z_n|\overline{\phi}_n \quad (57)$$

where

$$\beta_n = -\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{W}_{a,j}}\dot{\hat{W}}_{a,j} - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{b}_j}\dot{\hat{b}}_j \quad (58)$$

$$b_n^* = \max\{p_1^*, \ldots, p_n^*, \delta_1^*, \ldots, \delta_n^*\} \quad (59)$$

$$\overline{\phi}_n(\overline{x}_n) = \phi_n + \sum_{j=1}^{n-1}\left|\frac{\partial\alpha_{n-1}}{\partial x_j}\right|\phi_j + 1 + \sum_{j=1}^{n-1}\left|\frac{\partial\alpha_{n-1}}{\partial x_j}\right|. \quad (60)$$

Let

$$W_{a,n}^* = [1, g_1, \ldots, g_{n-1}, W_n^{*T}, W_1^{*T}, \ldots, W_{n-1}^{*T}]^T \quad (61)$$

$$\psi_{a,n} = \left[\beta_n, -\frac{\partial\alpha_{n-1}}{\partial x_1}x_2, \ldots, -\frac{\partial\alpha_{n-1}}{\partial x_{n-1}}x_n, \psi_n^T, -\frac{\partial\alpha_{n-1}}{\partial x_1}\psi_1^T, \ldots, -\frac{\partial\alpha_{n-1}}{\partial x_{n-1}}\psi_{n-1}^T\right]^T \quad (62)$$

then, (57) can be rewritten as

$$z_n\dot{z}_n \le z_n\left[g_n\beta(x)u + W_{a,n}^{*T}\psi_{a,n}\right] + b_n^*|z_n|\overline{\phi}_n. \quad (63)$$

Let the control input be designed as

$$u = \frac{N(\zeta_n)}{\beta(x)}\left(z_n + \hat{W}_{a,n}^T\psi_{a,n} + \hat{b}_n\overline{\phi}_n\tanh\left[\frac{z_n\overline{\phi}_n}{\epsilon_n}\right]\right) \quad (64)$$

$$N(\zeta_n) = \exp(\zeta_n^2)\cos\left(\frac{\pi}{2}\zeta_n\right) \quad (65)$$

$$\dot{\zeta}_n = z_n^2 + z_n\hat{W}_{a,n}\psi_{a,n} + z_n\hat{b}_n\overline{\phi}_n\tanh\left[\frac{z_n\overline{\phi}_n}{\epsilon_n}\right] \quad (66)$$

where ϵ_n is a small positive constant. For parameter adaptation, we introduce the following adaptation laws:

$$\dot{\hat{W}}_{a,n} = \Gamma_n\psi_a z_n - \Gamma_n\sigma_n\left(\hat{W}_{a,n} - W_{a,n}^0\right) \quad (67)$$

$$\dot{\hat{b}}_n = \lambda_n z_n\overline{\phi}_n\tanh\left[\frac{z_n\overline{\phi}_n}{\epsilon_n}\right] - \lambda_n\sigma_{b_n}(\hat{b}_n - b_n^0) \quad (68)$$

where $\Gamma_n = \Gamma_n^T > 0$, $\lambda_n > 0$, $\sigma_{W_n} > 0$, $\sigma_{b_n} > 0$ and $W_{a,n}^0$, $b_n^0 > 0$ are design constants.

Consider the Lyapunov function candidate

$$V_n = \tfrac{1}{2}z_n^2 + \tfrac{1}{2}\left(\hat{W}_{a,n} - W_{a,n}^*\right)^T\Gamma_n^{-1}\left(\hat{W}_{a,n} - W_{a,n}^*\right) + \frac{1}{2\lambda_n}(\hat{b}_n - b_n^*)^2. \quad (69)$$

Similarly, the time derivative of V_n satisfies

$$\begin{aligned}\dot{V}_n \le& -z_n^2 + g_nN(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n - \tfrac{1}{2}\sigma_{W_n}\left|\hat{W}_{a,n} - W_{a,n}^*\right|^2 \\ &- \tfrac{1}{2}\sigma_{b_n}(\hat{b}_n - b_n^*)^2 + b_n^*0.2785\epsilon_n \\ &+ \tfrac{1}{2}\sigma_{W_{a,n}}\left|W_{a,n}^* - W_{a,n}^0\right|^2 + \tfrac{1}{2}\sigma_{b_n}(b_n^* - b_n^0)^2 \\ \le& -\tfrac{3}{4}z_n^2 + g_nN(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n - \tfrac{1}{2}\sigma_{W_n}\left|\hat{W}_{a,n} - W_{a,n}^*\right|^2 \\ &- \tfrac{1}{2}\sigma_{b_n}(\hat{b}_n - b_n^*)^2 + b_n^*0.2785\epsilon_n \\ &+ \tfrac{1}{2}\sigma_{W_n}\left|W_{a,n}^* - W_{a,n}^0\right|^2 + \tfrac{1}{2}\sigma_{b_n}(b_n^* - b_n^0)^2. \end{aligned} \quad (70)$$

This yields

$$0 \le V_n(t) \le \rho_n + V_n(0) + e^{-C_{n1}t}\int_0^t (g_nN(\zeta_n)+1)\dot{\zeta}_n e^{C_{n1}\tau}\,d\tau \quad (71)$$

where $\rho_n := C_{n2}/C_{n1}$, the constants $C_{n1} > 0$ and $C_{n2} > 0$ are defined as

$$C_{n1} := \min\left\{\frac{3}{2}, \frac{\sigma_{W_n}}{\lambda_{\max}(\Gamma_n^{-1})}, \sigma_{b_n}\lambda_n\right\} \quad (72)$$

$$C_{n2} := b_n^*0.2785\epsilon_n + \tfrac{1}{2}\sigma_{W_n}\left\|W_{a,n}^* - W_{a,n}^0\right\|^2 + \tfrac{1}{2}\sigma_{b_n}(b_n^* - b_n^0)^2. \quad (73)$$

Using Lemma 1, we can conclude that $\zeta_n(t)$ and $V_n(t)$, hence $z_n(t)$, $\hat{W}_{a,n}(t)$, and $\hat{b}_{a,n}(t)$ are uniformly ultimately bounded. From the boundedness of $z_n(t)$, the boundedness of the extra term $\int_0^t g_{n-1}^2 z_n^2\, e^{-C_{n-1,1}(t-\tau)}\,d\tau$ at Step $n-1$ is readily obtained. Applying Lemma 1 $(n-1)$ times backward, it can be seen from the above design procedures that $V_i(t)$, $Z_i(t)$, $\hat{W}_{a,i}(t)$, $\hat{b}_{a,i}(t)$, and, hence, $x_i(t)$ are uniformly ultimately bounded.

The following theorem shows the stability and control performance of the closed-loop adaptive system.

Theorem 3.1: For perturbed strict feedback nonlinear system (1) with completely unknown virtual control coefficients g_i, $i = 1, \ldots, n$, under Assumptions 2.1 and 2.2, if we apply the control design procedure in the above statement, the solutions of the resulting closed-loop adaptive system are uniformly ultimately bounded.

Furthermore, given any $\mu^* > \sqrt{\sum_{i=1}^n 2\rho_i}$, there exists T such that, for all $t \ge T$, we have $|z(t)| \le \mu^*$. The compact set $\Omega_z = \{z \in R^n: |z(t)| \le \mu^*\}$ can be made as small as desired by an appropriate choice of the design constants. Correspondingly, the output $y(t)$ satisfies the following property:

$$|y(t)| \le \sqrt{2(\rho_1 + C_1) + 2V_1(0)e^{-C_{11}t}} \quad (74)$$

where $\rho_i := C_{i2}/C_{i1}$, $i = 1, \ldots, n$, C_1 is the upper bound of $\int_0^t(\dot{\zeta}_1 + g_1^2z_2^2 + g_1N(\zeta_1)\dot{\zeta}_1)\, e^{-C_{11}(t-\tau)}\,d\tau$, and constants $C_{i1} > 0$ and $C_{i2} > 0$ are defined by (55) and (56), respectively.

Proof: The proof can be easily completed by following the above design procedures from Step 1 to Step n. Since $y(t) = x_1(t) = z_1(t)$, from the definition of V_1 and (36), the property (74) can be easily obtained. Thus, by appropriately choosing the design constants, we can achieve the regulation of the output $y(t)$ to any prescribed accuracy while keeping the boundedness of all the signals and states of the closed-loop system. ◊

Remark 3.5: Decreasing ϵ_i, σ_{W_i} and σ_{b_i} will help to reduce the size of Ω_z. However, if ϵ_i, σ_{W_i} and σ_{b_i} are too small, it may not be enough to prevent the parameter estimates from drifting to very large values in the presence of the neural-network approximation errors, where the large $\hat{W}_i$ might result in a variation of a high-gain control. Therefore, in practical applications, the design parameters should be adjusted carefully for achieving suitable transient performance and control action.

Remark 3.6: Compared with the works in [6], [7], [17], and [19], the proposed robust adaptive controller in this paper can cope with the parametric uncertainty and bounding uncertainty, as well as the unknown virtual control coefficients. The unknown system functions $f_i(\overline{x}_i)$ are approximated by neural networks. The unknown bounds of neural approximation errors are also adaptively tuned. The proposed design method expands the class of nonlinear systems to which robust adaptive control approaches can be applied due to the employment of online linearly parameterized approximator.

IV. Simulation

To illustrate the proposed robust adaptive control algorithms, we consider the regulation of the second-order system

$$\begin{aligned} \dot{x}_1 &= g_1 x_2 + f_1(x_1) + \Delta_1(t, x) \\ \dot{x}_2 &= g_2 u + f_2(x) + \Delta_2(t, x) \\ y &= x_1 \end{aligned} \tag{75}$$

where $x = [x_1, x_2]^T$, g_1, g_2 are unknown virtual control coefficients, $f_1(x), f_2(x)$ are unknown system functions, and $\Delta_1(t, x), \Delta_2(t, x)$ are unknown bounded disturbances. For simulation purpose, we assume that $g_1 > 0$, $g_2 > 0$ and let

$$f_1(x_1) = 0.1x_1^2 \tag{76}$$

$$f_2(x) = 0.2e^{-x_2} + x_1\sin(x_2) \tag{77}$$

$$\Delta_1(t, x) = 0.6\sin(x_2) \tag{78}$$

$$\Delta_2(t, x) = 0.5(x_1^2 + x_2^2)\sin^3 t \tag{79}$$

and $g_1 = 1$ and $g_2 = 1$. The bounds on Δ_1 and Δ_2 are

$$|\Delta_1(x, t)| \le p_1^*\phi_1(x_1) \tag{80}$$

$$|\Delta_2(x, t)| \le p_2^*\phi_2(x) \tag{81}$$

where $p_1^* := 0.6, p_2^* := 0.5$, $\phi_1(x_1) = 1$, and $\phi_2(x) = x_1^2 + x_2^2$. We use RBF neural networks to approximate $f_1(x_1)$, $f_2(x)$, i.e., $f_1(x_1) = W_1^{*T}\psi_1(x_1) + \omega_1(x_1)$, $f_2(x) = W_2^{*T}\psi_2(x) + \omega_2(x)$, where $|\omega_1| \le \delta_1^*$, $|\omega_2| \le \delta_2^*$. $b_1^* = \max\{\delta_1^*, p_1^*\}$, $b_2^* = \max\{\delta_1^*, \delta_1^*, p_1^*, p_2^*\}$. For the design of robust adaptive controller, let $\hat{W}_{a,1}, \hat{W}_{a,2}, \hat{b}_1, \hat{b}_2$ be the estimates of unknown parameters $W_{a,1}^* = W_1^*$, $W_{a,2}^* = [1, g_1, W_2^{*T}, W_1^{*T}]$, b_1^*, b_2^*, and $z_1 = x_1$, $z_2 = x_2 - \alpha_1$, we have

$$\overline{\phi}_1 = 1 + \phi_1 \tag{82}$$

$$\alpha_1 = N(\zeta_1)\left(z_1 + \hat{W}_{a,1}^T\psi_{a,1}(x_1) + \hat{b}_1\overline{\phi}_1(x_1) \cdot \tanh\left[\frac{x_1\overline{\phi}_1(x_1)}{\epsilon_1}\right]\right) \tag{83}$$

$$\overline{\phi}_2 = \phi_2 + 1 + \left|\frac{\partial\alpha_1}{\partial x_1}\right|(\phi_1 + 1) \tag{84}$$

$$u = N(\zeta_2)\left(z_2 + \hat{W}_{a,2}^T\psi_{a,2} + \hat{b}_2\overline{\phi}_2\tanh\left[\frac{z_2\overline{\phi}_2}{\epsilon_2}\right]\right) \tag{85}$$

where $N(\zeta_i) = \exp(\zeta_i^2)\cos((\pi/2)\zeta_i)$, $i = 1, 2$ are the Nussbaum functions, $\psi_{a,1} = \psi_1$

$$\psi_{a,2} = \left[-\frac{\partial\alpha_1}{\partial\hat{W}_{a,1}}\dot{\hat{W}}_{a,1} - \frac{\partial\alpha_1}{\partial\hat{b}_1}\dot{\hat{b}}_1, -\frac{\partial\alpha_1}{\partial x_1}x_2, \psi_2^T, -\frac{\partial\alpha_1}{\partial x_1}\psi_1^T\right]^T$$

and ζ_1, ζ_2 are computed using (22) and (66), respectively.

Parameters update laws

$$\dot{\hat{W}}_{a,1} = \Gamma_1 z_1\psi_{a,1} - \Gamma_1\sigma_{W_1}(\hat{W}_{a,1} - W_{a,1}^0) \tag{86}$$

$$\dot{\hat{W}}_{a,2} = \Gamma_2 z_2\psi_{a,2} - \Gamma_2\sigma_{W_2}(\hat{W}_{a,2} - W_{a,2}^0) \tag{87}$$

$$\dot{\hat{b}}_1 = \lambda_1 z_1\overline{\phi}_1\tanh\left[\frac{z_1\overline{\phi}_1}{\epsilon_1}\right] - \lambda_1\sigma_{b_1}(\hat{b}_1 - b_1^0) \tag{88}$$

$$\dot{\hat{b}}_2 = \lambda_2 z_2\overline{\phi}_2\tanh\left[\frac{z_2\overline{\phi}_2}{\epsilon_2}\right] - \lambda_2\sigma_{b_2}(\hat{b}_2 - b_2^0). \tag{89}$$

The selection of the centers and widths of RBF has a great influence on the performance of the adaptive neural controller. It has been indicated [20] that Gaussian RBF NNs arranged on a regular lattice on R^n can uniformly approximate sufficiently smooth functions on closed bounded subsets. Accordingly, in the following simulation studies, we select the centers and widths as: Neural network $W_1^{*T}\psi_1(x_1)$ contains nine nodes, with centers μ_l ($l = 1, \ldots, 9$) evenly spaced in $[-5, 5]$, and widths $\eta_l = 1$ ($l = 1, \ldots, 9$). Neural network $W_2^{*T}\psi_2(x)$ contains 63 nodes, with centers μ_l ($l = 1, \ldots, 63$) evenly spaced in $[-5, 5] \times [-7.5, 7.5]$, and widths $\eta_l = 1$ ($l = 1, \ldots, 63$). The following initial conditions and controller design parameters are adopted in the simulation: $x(0) = [-0.5, 0]^T$, $\hat{W}_{a,1}(0) = 0$, $\hat{W}_{a,2}(0) = 0$, $\hat{b}_1(0) = 0$, $\hat{b}_2(0) = 0$, and $\Gamma_1 = \Gamma_2 = 0.5$, $\lambda_1 = \lambda_2 = 0.1$, $\sigma_{W_1} = \sigma_{W_2} = \sigma_{b_1} = \sigma_{b_1} = 0.1$, $\epsilon_1 = \epsilon_2 = 0.1$, $W_{a,1}^0 = W_{a,2}^0 = 0$, and $b_1^0 = b_2^0 = 0.1$.

Simulation results in Figs. 1–6 show the effectiveness of the proposed robust adaptive control design for system (75) with uncertainties and completely unknown g_i, $i = 1, 2$. Fig. 1 shows that the system output converges to a small neighborhood around zero. The boundedness of control input is shown in Fig. 2. The boundedness of weights $\hat{W}_{a,1}$, $\hat{W}_{a,2}$ as well as the parameter estimates $\hat{b}_1$ and $\hat{b}_2$ are illustrated in Figs. 3 and 4, respectively. Figs. 5 and 6 show the variations of Nussbaum gains $N(\zeta_1)$, $N(\zeta_2)$ and parameters ζ_1, ζ_2 respectively, which are also bounded.

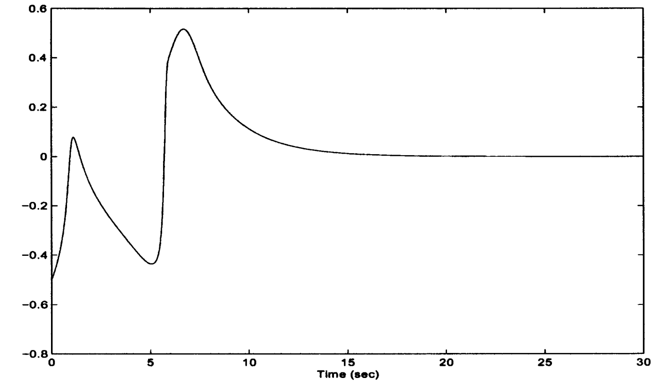


Fig. 1. Output y.

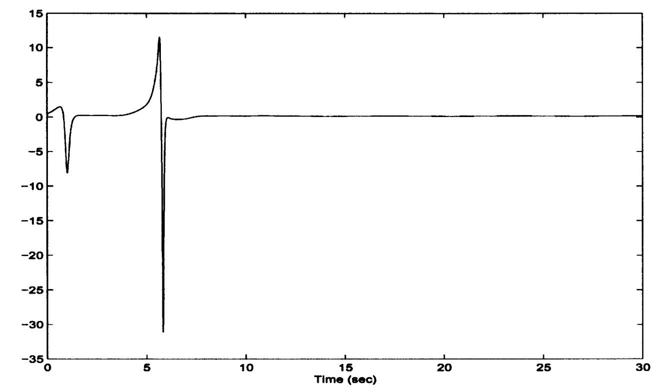


Fig. 2. Control input u.

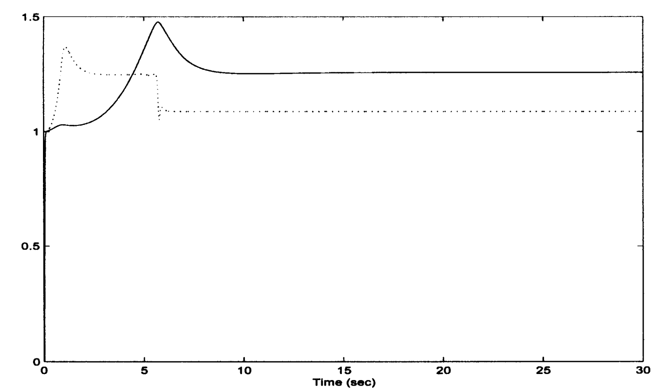


Fig. 3. Boundedness of weights $\|\hat{W}_{a,1}\|$: "solid line." $\|\hat{W}_{a,2}\|$: "dashdot line".

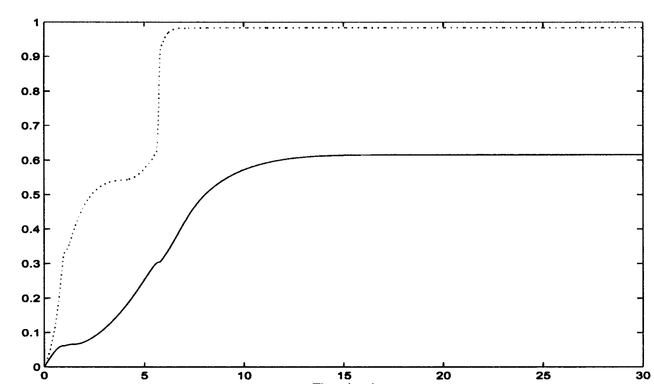


Fig. 4. Boundedness of parameters $\hat{b}_1$: "solid line." $\hat{b}_2$: "dashdot line."

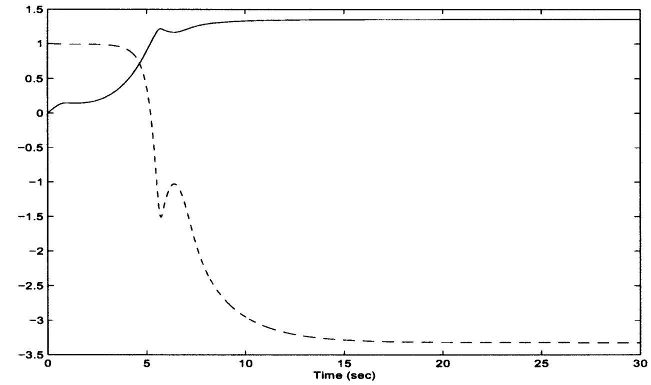


Fig. 5. Adapting parameters ζ_1: "solid line" and "gain" $N(\zeta_1)$: "dash line."

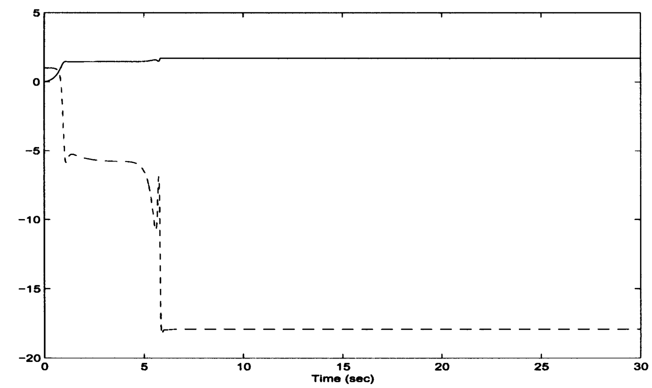


Fig. 6. Adapting parameters ζ_2: "solid line" and "gain." $N(\zeta_2)$: "dash line."

V. CONCLUSION

In this paper, a robust adaptive control design for a class of perturbed uncertain strict feedback nonlinear systems with unknown virtual control coefficients has been presented. With the utilization of iterative Lyapunov and neural approximators, the proposed design method expands the class of nonlinear systems for which robust adaptive control approaches have been studied. The design method do not require the *a priori* knowledge of the signs of the unknown virtual control coefficients due to the incorporation of Nussbaum gain in the controller design. It has been proven that the proposed robust adaptive scheme can guarantee the uniform ultimate boundedness of the closed-loop system signals. The control performance can be guaranteed by an appropriate choice of the design parameters. Simulation results have shown the effectiveness of the proposed method.

APPENDIX
PROOF OF LEMMA 1

Rewrite (5) as

$$\begin{aligned} 0 &< V(t) \\ &\le C_0 + g_1 \int_0^t N(\zeta)\dot{\zeta} e^{-C_1(t-\tau)}\, d\tau + \left| \int_0^t \dot{\zeta}(t) e^{-C_1(t-\tau)}\, d\tau \right| \\ &\le C_0 + |g_1| \int_0^t \left| N(\zeta)\dot{\zeta} \right| \, d\tau + \int_0^t \left| \dot{\zeta}(t) \right| \, d\tau, \quad \forall\, t \in [0,\, t_f). \end{aligned} \tag{90}$$

Depending on the signs of $N(\zeta)$ and $\dot{\zeta}(t)$, (90) can be further written as

$$0 < V(t) \le C \pm |g_1| \int_0^{\zeta(t)} N(\zeta)\, d\zeta \pm \zeta(t), \qquad \forall t \in [0, t_f) \tag{91}$$

where constant $C = C_0 \mp \zeta(0)$.

We first show that $\zeta(t)$ is bounded on $[0, t_f)$ by seeking a contradiction. Two cases are considered.

Case 1: Suppose that $\zeta(t)$ has no upper bounds on $[0, t_f)$. The properties of Nussbaum-type function ensure the existence of two monotonely increasing sequences $\{\zeta_n^{(j)}\}$ with $\zeta_1^{(j)} > |\zeta(0)|$ and $\lim_{n\to\infty} \zeta_n^{(j)} = \infty$, $j = 1, 2$, such that

$$\lim_{n\to\infty} \frac{1}{\zeta_n^{(1)}} \int_0^{\zeta_n^{(1)}} N(s)\, ds = \infty \tag{92}$$

$$\lim_{n\to\infty} \frac{1}{\zeta_n^{(2)}} \int_0^{\zeta_n^{(2)}} N(s)\, ds = -\infty. \tag{93}$$

Since $\zeta(t)$ has no upper bounds on $[0, t_f)$, thus, there exist two monotonely increasing sequences $\{t_n^{(j)}\}$, $j = 1, 2$, such that $\zeta(t_n^{(j)}) = \zeta_n^{(j)}$, $j = 1, 2$. Clearly, $\lim_{n\to\infty} t_n^{(j)} = t_f$, $j = 1, 2$. Dividing (91) by $\zeta(t_n^{(j)}) = \zeta_n^{(j)} > 0$, we obtain

$$0 < \frac{V\left(t_n^{(1)}\right)}{\zeta\left(t_n^{(1)}\right)} \le \frac{C}{\zeta_n^{(1)}} \pm \frac{|g_1|}{\zeta_n^{(1)}} \int_0^{\zeta_n^{(1)}} N(\zeta)\, d\zeta \pm 1 \tag{94}$$

$$0 < \frac{V\left(t_n^{(2)}\right)}{\zeta\left(t_n^{(2)}\right)} \le \frac{C}{\zeta_n^{(2)}} \pm \frac{|g_1|}{\zeta_n^{(2)}} \int_0^{\zeta_n^{(2)}} N(\zeta)\, d\zeta \pm 1 \tag{95}$$

where $n = 1, 2, \ldots$. Noting that $t_n^{(j)} \to t_f$, $\zeta_n^{(j)} \to \infty$, $j = 1, 2$ when $n \to \infty$. Thus, if $N(\zeta) > 0$, (95) contradicts (93), and when $N(\zeta) < 0$, (94) contradicts (92). Therefore, $\zeta(t)$ is upper bounded on $[0, t_f)$.

Case 2: Suppose that $\zeta(t)$ has no lower bounds on $[0, t_f)$. Thus, there exist two monotonely increasing sequences $\{t_n^{(j)}\}$, $j = 1, 2$, such that $\zeta(t_n^{(j)}) = -\zeta_n^{(j)}$, $j = 1, 2$. Clearly, $\lim_{n\to\infty} t_n^{(j)} = t_f$, $j = 1, 2$. Since the function $N(\cdot)$ is even, (91) can be further written as

$$0 < V(t) \le C \mp |g_1| \int_0^{-\zeta(t)} N(\zeta)\, d\zeta \pm \zeta(t), \quad \forall t \in [0, t_f). \tag{96}$$

Dividing (96) by $-\zeta(t_n^{(j)}) = \zeta_n^{(j)} > 0$, we obtain

$$0 < -\frac{V\left(t_n^{(1)}\right)}{\zeta\left(t_n^{(1)}\right)} \le \frac{C}{\zeta_n^{(1)}} \mp \frac{|g_1|}{\zeta_n^{(1)}} \int_0^{\zeta_n^{(1)}} N(\zeta)\, d\zeta \mp 1 \tag{97}$$

$$0 < -\frac{V\left(t_n^{(2)}\right)}{\zeta\left(t_n^{(2)}\right)} \le \frac{C}{\zeta_n^{(2)}} \mp \frac{|g_1|}{\zeta_n^{(2)}} \int_0^{\zeta_n^{(2)}} N(\zeta)\, d\zeta \mp 1 \tag{98}$$

where $n = 1, 2, \ldots$. Noting that $t_n^{(j)} \to t_f$, $\zeta_n^{(j)} \to \infty$, $j = 1, 2$ when $n \to \infty$. Similarly, there is a contradiction no matter what the sign of $N(\zeta)$ is. Therefore, $\zeta(t)$ is lower bounded on $[0, t_f)$.

Thus, we conclude that the boundedness of $\zeta(t)$ on $[0, t_f)$. As an immediate result, $V(t)$ and $\int_0^t g_1 N(\zeta)\dot{\zeta}\, d\tau$ are also bounded on $[0, t_f)$. $\diamondsuit$

References

[1] Z. Ding, "Adaptive control of nonlinear systems with unknown virtual control coefficients," *Int. J. Adaptive Contr. Signal Processing*, vol. 14, pp. 505–517, 2000.

[2] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*. Boston, MA: Kluwer, 2002.

[3] S. S. Ge, C. C. Hang, and T. Zhang, "A direct method for robust adaptive nonlinear control with guaranteed transient performance," *Syst. Contr. Lett.*, vol. 37, pp. 275–284, 1999.

[4] A. Ilchmann, *Non-Identifier-Based High-Gain Adaptive Control*. New York: Springer-Verlag, 1993.

[5] A. Isidori, *Nonlinear Control Systems*. Berlin, Germany: Springer-Verlag, 1995.

[6] Z. P. Jiang and D. J. Hill, "A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics," *IEEE Trans. Automat. Contr.*, vol. 44, pp. 1705–1711, Sept. 1999.

[7] Z. P. Jiang and L. Praly, "Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties," *Automatica*, vol. 34, pp. 825–840, 1998.

[8] J. Kaloust and Z. Qu, "Continuous robust control design for nonlinear uncertain systems without *a priori* knowledge of control direction," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 276–282, Feb. 1995.

[9] I. Kanellakopoulos, P. V. Kokotovic, and R. Marino, "An extended direct scheme for robust adaptive nonlinear control," *Automatica*, vol. 27, pp. 247–255, 1991.

[10] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 1241–1253, July 1991.

[11] H. K. Khalil, *Nonlinear Systems*. New York: MacMillan, 1996.

[12] E. B. Kosmatopoulos, M. M. Polycarpou, M. A. Christodoulou, and P. A. Ioannou, "High-order neural network structures for identification of dynamical systems," *IEEE Trans. Neural Networks*, vol. 6, pp. 422–431, Mar. 1995.

[13] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.

[14] B. Martensson, "Remarks on adaptive stabilization of first-order nonlinear systems," *Syst. Control Lett.*, vol. 14, pp. 1–7, 1990.

[15] D. R. Mudgett and A. S. Morse, "Adaptive stabilization of linear systems with unknown high frequency gain," *IEEE Trans. Automat.Contr.*, vol. AC-30, pp. 549–554, Feb. 1985.

[16] R. D. Nussbaum, "Some remarks on the conjecture in parameter adaptive control," *Syst. Contr. Lett.*, vol. 3, pp. 243–246, 1983.

[17] M. M. Plycarpou and P. A. Ioannou, "A robust adaptive nonlinear control design," *Automatica*, vol. 31, pp. 423–427, 1995.

[18] M. M. Plycarpou and M. Mears, "Stable adaptive tracking of uncertain systems using nonlinearly parameterized online approximators," *Int. J. Contr.*, vol. 70, pp. 363–384, 1998.

[19] M. M. Polycarpou, "Stable adaptive neural control scheme for nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 447–451, Feb. 1996.

[20] R. M. Sanner and J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Networks*, vol. 3, pp. 837–863, May 1992.

[21] S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. Automat. Contr.*, vol. 34, pp. 1123–1131, June 1989.

[22] D. Seto, A. M. Annaswamy, and J. Baillieul, "Adaptive control of nonlinear systems with triangular structure," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 1411–1428, Aug. 1994.

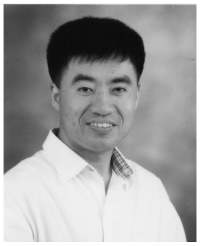
[23] J. T. Spooner and K. M. Passino, "Stable adaptive control using fuzzy systems and neural networks," *IEEE Trans. Fuzzy Syst.*, vol. 4, pp. 339–359, 1996.

[24] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1994.

[25] B. Yao and M. Tomizuka, "Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form," *Automatica*, vol. 33, pp. 893–900, 1997.

[26] X. Ye and J. Jiang, "Adaptive nonlinear design without *a priori* knowledge of control directions," *IEEE Trans. Automat. Contr.*, vol. 43, pp. 1617–1621, Sept. 1998.

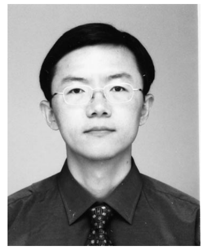
[27] A. Yesildirek and F. L. Lewis, "Feedback linearization using neural networks," *Automatica*, vol. 31, pp. 1659–1664, 1995.



Shuzhi Sam Ge (S'90–M'92–SM'00) received the B.Sc. degree from Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China, in 1986, and the Ph.D. degree and the Diploma of Imperial College (DIC) degree from Imperial College of Science, Technology and Medicine, University of London, London, U.K., in 1993.

From 1992 to 1993, he did his postdoctoral research at Leicester University, Leicester, U.K. He has been with the Department of Electrical and Computer Engineering, the National University of Singapore, Singapore, since 1993, and is currently an Associate Professor. He visited the Laboratoire de'Automatique de Grenoble, Grenoble, France in 1996, the University of Melbourne, Melbourne, Australia in 1998 and 1999, and University of Petroleum, Shanghai Jiaotong University, Shanghai, China, in 2001. He serves as a technical consultant to local industry. He has authored and coauthored more than 100 international journal and conference papers, two monographs, and is a co-holder of two patents. His current research interests are control of nonlinear systems, neural networks and fuzzy logic, robot control, real-time implementation, path planning, and sensor fusion.

Dr. Ge served as an Associate Editor on the Conference Editorial Board of the IEEE Control Systems Society in 1998 and 1999, has been serving as an Associate Editor, IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY since 1999, and a Member of the Technical Committee on Intelligent Control of the IEEE Control System Society since 2000. He was the recipient of the 1999 National Technology Award, 2001 University Young Research Award, and 2002 Temasek Young Investigator Award, Singapore.



Jing Wang (M'01) received the B.Eng. degree and the Ph.D. degree in control theory and control engineering, both from Central South University of Technology, Changsha, China, in 1992 and 1997, respectively.

He was a Postdoctoral Research Fellow at the Institute of Computing Technology, Chinese Academy of Sciences, from 1997 to 1999, and at the National University of Singapore, Singapore, from 1999 to 2002. He visited the Hong Kong Polytechnic University as a Research Associate in 1998. Since March 2002, he has been with School of Electrical and Computer Science of University of Central Florida, Orlando, as Postdoctoral Visiting Scholar. His current research interests include adaptive control of nonlinear systems, neural networks and fuzzy control, robot control and trajectory optimization, and control applications.

Dr. Wang is the corecipient of Best Theoretical Paper Award in 2002 at the Fourth World Congress on Intelligent Control and Automation, Shanghai, China.