

Adaptive Neural Network Tracking Control of MIMO Nonlinear Systems With Unknown Dead Zones and Control Directions

Tianping Zhang and Shuzhi Sam Ge, *Fellow, IEEE*

Abstract—In this paper, adaptive neural network (NN) tracking control is investigated for a class of uncertain multiple-input–multiple-output (MIMO) nonlinear systems in triangular control structure with unknown nonsymmetric dead zones and control directions. The design is based on the principle of sliding mode control and the use of Nussbaum-type functions in solving the problem of the completely unknown control directions. It is shown that the dead-zone output can be represented as a simple linear system with a static time-varying gain and bounded disturbance by introducing characteristic function. By utilizing the integral-type Lyapunov function and introducing an adaptive compensation term for the upper bound of the optimal approximation error and the dead-zone disturbance, the closed-loop control system is proved to be semiglobally uniformly ultimately bounded, with tracking errors converging to zero under the condition that the slopes of unknown dead zones are equal. Simulation results demonstrate the effectiveness of the approach.

Index Terms—Adaptive control, dead zone, neural network (NN) control, Nussbaum function, sliding mode control.

I. INTRODUCTION

IN recent years, adaptive control system design using either neural networks (NNs) or fuzzy logic systems to parametrize the unknown nonlinearities has received much attention [1]–[5]. Direct adaptive tracking control was proposed for a class of continuous-time nonlinear systems using radial basis function NNs in [1]. By utilizing modulation function and fuzzy systems, adaptive fuzzy control was investigated in [2]. Multi-layer-neural-network-based indirect adaptive control was developed for feedback linearization of a class of nonlinear systems in [3]. Using a families of novel integral Lyapunov functions for avoiding the possible controller singularity problem without using projection, adaptive neural controls were presented for a class of nonlinear systems in a Brunovsky form [4], and for a class of multiple-input–multiple-output (MIMO) nonlinear systems with a triangular structure in the control inputs in [5] and

[6]. Furthermore, two design schemes of adaptive controller for single-input–single-output (SISO)/MIMO uncertain nonlinear systems were proposed in [7] and [8], and ensured tracking error converging to zero. Based on the principle of sliding mode control and the approximation capability of fuzzy systems, decentralized indirect and direct adaptive fuzzy control schemes were proposed for a class of nonlinear systems with unknown constant control gains and unknown function control gains in [9] and [10].

When there is no *a priori* knowledge about the signs of control gains, adaptive control of such systems becomes much more difficult. The first solution was given in [11] for a class of first-order linear systems, where the Nussbaum-type gain was originally proposed. When the high-frequency control gains and their signs are unknown, gains of Nussbaum-type [11] have been effectively used in controller design in solving the difficulty of unknown control directions [12] and [13].

Nonsmooth nonlinear characteristics such as dead zone, backlash, and hysteresis are common in actuator and sensors such as mechanical connections, hydraulic actuators, and electric servomotors. Dead zone is one of the most important nonsmooth nonlinearities in many industrial processes, which can severely limit system performance; and its study has been drawing much interest in the control community for a long time [14]–[20]. To handle the systems with unknown dead zones, adaptive dead-zone inverses were proposed in [14]–[16]. In [14] and [15], adaptive dead-zone inverses were built for linear systems with unmeasurable dead-zone outputs. Asymptotical adaptive cancellation of an unknown dead zone was achieved analytically under the condition that the output of a dead zone was measurable in [16]. A compensation scheme was presented for general nonlinear actuator dead zone of unknown width in [18]. In [19], by a given matching condition to the reference model, adaptive control with adaptive dead-zone inverse has been introduced. By using a new description of a dead zone with equal slopes, robust adaptive control was developed for a class of nonlinear systems in [20] without constructing the inverse of the dead zone. In [21], decentralized variable structure control was proposed for a class of uncertain large scale linear systems with state time-delay and dead-zone input. However, the parameters u_{i-} , u_{i+} of the dead zones [21] need to be known, and the disturbances satisfy the matching condition. In [22], a servomechanism problem of controlling a scalar output variable to λ -track was addressed for a class of special SISO linear system with nonlinear actuator characteristics. Adaptive output feedback control using backstepping and smooth inverse

Manuscript received June 26, 2007; revised April 07, 2008 and July 29, 2008; accepted October 03, 2008. First published February 03, 2009; current version published February 27, 2009. This work was supported in part by the National Natural Science Foundation of China under Grants 60774017, 60874045, and 60428304.

T. P. Zhang is with the Department of Automation, College of Information Engineering, Yangzhou University, Yangzhou 225009, China (e-mail: tpzhang@yzu.edu.cn).

S. S. Ge is with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576, Singapore (e-mail: samge@nus.edu.sg).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TNN.2008.2010349

function of the dead zone was proposed for a class of SISO nonlinear systems with unknown dead zone [23].

In this paper, we consider a class of uncertain MIMO nonlinear systems with both unknown dead zones and unknown gain signs. By employing analytic extension for the functions outside the dead band and introducing characteristic functions, the dead-zone output is represented as a simple linear system with a static time-varying gain and bounded disturbance. Based on the proposed novel description of dead-zone model, robust adaptive NN control is developed without necessarily constructing a dead-zone inverse and knowing some parameter bounds of dead zones. In addition, the problem of both unknown control direction and unknown control gain functions is solved by using Nussbaum-type functions and employing integral-type Lyapunov function. Moreover, all signals involved in the closed-loop system are proved to be semiglobally uniformly ultimately bounded, with tracking errors converging to zero under the condition that the slopes of unknown dead zones are equal.

This paper is organized as follows. The problem formulation and preliminaries are given in Section II. Section III presents multilayer NNs used in the controller design, and an adaptive NN control is developed for SISO systems using an integral-type Lyapunov function. Furthermore, this scheme is extended to MIMO systems. The closed-loop system stabilities are analyzed as well. In Section IV, adaptive NN control is investigated for a class of nonlinear MIMO systems with the dead zones of equal slopes. Simulation results are performed to demonstrate the effectiveness of the approach in Section V. Section VI contains the conclusions.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

Consider a class of uncertain MIMO nonlinear systems in triangular control structure with dead zones in the following form:

$$\text{Plant:} \quad \begin{cases} \dot{x}_{1j} = x_{1,j+1}, & j=1, \dots, n_1 - 1 \\ \dot{x}_{1n_1} = f_1(x) + b_1(x_1)u_1 \\ \dot{x}_{ij} = x_{i,j+1}, & j=1, \dots, n_i - 1 \\ \dot{x}_{in_i} = f_i(x, u_1, \dots, u_{i-1}) \\ \quad + b_i(x_1, \dots, x_i)u_i, & i=2, \dots, m \\ y_1 = x_{11}, \dots, y_m = x_{m1} \end{cases} \quad (1)$$

$$\text{Dead zone:} \quad u_i = D_i(v_i) = \begin{cases} g_{ir}(v_i), & \text{if } v_i \geq b_{ir} \\ 0, & \text{if } b_{il} < v_i < b_{ir} \\ g_{il}(v_i), & \text{if } v_i \leq b_{il} \end{cases} \quad (2)$$

where $x = [x_1^T, x_2^T, \dots, x_m^T]^T \in R^n$ is the state vector $x_i = [x_{i1}, \dots, x_{in_i}]^T$, $i = 1, \dots, m$, $n = \sum_{i=1}^m n_i$; $f_1(x), f_2(x, u_1), \dots, f_m(x, u_1, \dots, u_{m-1})$ are the unknown continuous functions; $b_1(x_1), b_2(\bar{x}_2), \dots, b_m(\bar{x}_m)$ are the unknown differentiable control gains $\bar{x}_i = [x_1^T, x_2^T, \dots, x_i^T]^T$; $y_i \in R$ denotes the i th subsystem output; $u_i \in R$ is the output of the i th dead zone (and the input to the i th subsystem); $v_i(t) \in R$ is the input to the i th dead zone; b_{il} and b_{ir} are the unknown parameters of the i th dead zone; and the functions

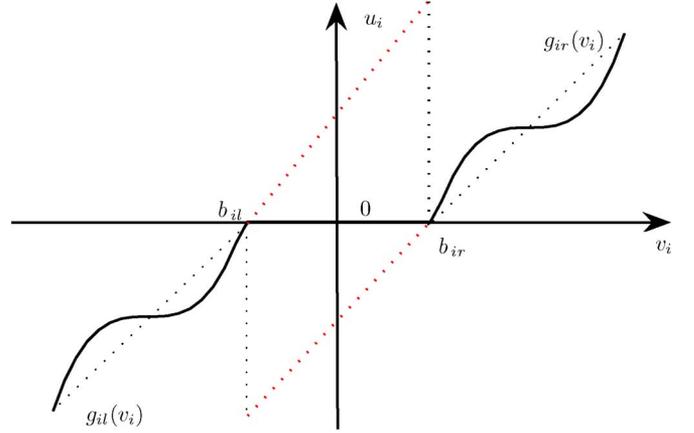


Fig. 1. Nonsymmetric nonlinear dead-zone model.

$g_{ir}(v_i)$ and $g_{il}(v_i)$ are smooth nonlinear functions. The dead zone with the input v_i is shown in Fig. 1. This describes a very general class of $D_i(v_i)$. As $g_{ir}(v_i)$, $g_{il}(v_i)$, b_{ir} , and b_{il} are all unknown, control system design becomes difficult.

To facilitate control system design, we need to make the following assumptions that first appeared in [24].

Assumption 1: The dead-zone outputs u_1, \dots, u_m are not available.

Assumption 2: The dead-zone parameters b_{ir} and b_{il} are unknown bounded constants, but their signs are known, i.e., $b_{ir} > 0$ and $b_{il} < 0$, $i = 1, \dots, m$.

Assumption 3: The functions $g_{il}(v_i)$ and $g_{ir}(v_i)$ are smooth, and there exist unknown positive constants k_{il0} , k_{il1} , k_{ir0} , and k_{ir1} such that

$$0 < k_{il0} \leq g'_{il}(v_i) \leq k_{il1} \quad \forall v_i \in (-\infty, b_{il}] \quad (3)$$

$$0 < k_{ir0} \leq g'_{ir}(v_i) \leq k_{ir1} \quad \forall v_i \in [b_{ir}, +\infty) \quad (4)$$

where $g'_{il}(v_i) = dg_{il}(z)/dz|_{z=v_i}$ and $g'_{ir}(v_i) = dg_{ir}(z)/dz|_{z=v_i}$.

For notational convenience, we will extend the definition for functions $g_{il}(v_i)$ and $g_{ir}(v_i)$ as follows:

$$g_{il}(v_i) = g'_{il}(b_{il})(v_i - b_{il}) \quad \forall v_i \in (b_{il}, b_{ir}] \quad (5)$$

$$g_{ir}(v_i) = g'_{ir}(b_{ir})(v_i - b_{ir}) \quad \forall v_i \in [b_{il}, b_{ir}). \quad (6)$$

Therefore, (3) is true for $v_i \in (-\infty, b_{ir}]$, and (4) is also true for $v_i \in [b_{il}, +\infty)$.

According to the differential mean value theorem, we know that there exist $\xi_{il}(v_i) \in (-\infty, b_{il})$ and $\xi_{ir}(v_i) \in (b_{ir}, +\infty)$ such that

$$\begin{aligned} g_{il}(v_i) &= g_{il}(v_i) - g_{il}(b_{il}) \\ &= g'_{il}(\xi_{il}(v_i))(v_i - b_{il}) \quad \forall v_i \in (-\infty, b_{il}] \quad (7) \end{aligned}$$

$$\begin{aligned} g_{ir}(v_i) &= g_{ir}(v_i) - g_{ir}(b_{ir}) \\ &= g'_{ir}(\xi_{ir}(v_i))(v_i - b_{ir}) \quad \forall v_i \in [b_{ir}, +\infty). \quad (8) \end{aligned}$$

Based on Assumption 3, the dead zone (2) can be rewritten as follows:

$$u_i = K_i^T(v_i(t)) \Phi_i(v_i(t)) v_i(t) + d_i(v_i(t)) \quad \forall t \geq 0 \quad (9)$$

where $|d_i(v_i(t))| \leq p_i^*$, p_i^* is an unknown positive constant with $p_i^* = (k_{ir1} + k_{ki1}) \max\{b_{ir}, -b_{il}\}$, and

$$K_i(v_i(t)) = [K_{ir}(v_i(t)), K_{il}(v_i(t))]^T \quad (10)$$

$$K_{ir}(v_i(t)) = \begin{cases} 0, & \text{if } v_i(t) \leq b_{il} \\ g'_{ir}(b_{ir}), & \text{if } b_{il} < v_i(t) < b_{ir} \\ g'_{ir}(\xi_{ir}(v_i(t))), & \text{if } b_{ir} \leq v_i(t) < +\infty \end{cases} \quad (11)$$

$$K_{il}(v_i(t)) = \begin{cases} g'_{il}(\xi_{il}(v_i(t))), & \text{if } -\infty < v_i(t) \leq b_{il} \\ g'_{il}(b_{il}), & \text{if } b_{il} < v_i(t) < b_{ir} \\ 0, & \text{if } v_i(t) \geq b_{ir} \end{cases} \quad (12)$$

$$\Phi_i(v_i(t)) = [\varphi_{ir}(v_i(t)), \varphi_{il}(v_i(t))]^T \quad (13)$$

$$\varphi_{ir}(v_i(t)) = \begin{cases} 1, & \text{if } v_i(t) > b_{il} \\ 0, & \text{if } v_i(t) \leq b_{il} \end{cases} \quad (14)$$

$$\varphi_{il}(v_i(t)) = \begin{cases} 1, & \text{if } v_i(t) < b_{ir} \\ 0, & \text{if } v_i(t) \geq b_{ir} \end{cases} \quad (15)$$

$$d_i(v_i(t)) = \begin{cases} -g'_{ir}(\xi_{ir}(v_i(t)))b_{ir}, & \text{if } v_i(t) \geq b_{ir} \\ -[g'_{il}(b_{il}) + g'_{ir}(b_{ir})]v_i(t), & \text{if } b_{il} < v_i(t) < b_{ir} \\ -g'_{il}(\xi_{il}(v_i(t)))b_{il}, & \text{if } v_i(t) \leq b_{il}. \end{cases} \quad (16)$$

Remark 1: For the case of linear dead zone outside the dead-band, control system design has been extensively studied [14], [15], [20], [25]. To the best of our knowledge, (9) is to capture the most realistic situation, and as such it is different from the existing idealized description [14], [16], [17], [19], [20], [23]. As shown in [24], we know that $K_i^T(v_i(t))\Phi_i(v_i(t)) \in [\min\{k_{i0}, k_{ir0}\}, k_{i1} + k_{ir1}] \subset (0, +\infty)$.

The control objective is to design an adaptive controller $v_i(t)$ for system (1) such that the output y_i follows the specified desired trajectory y_{id} , $i = 1, \dots, m$.

Define x_{id} and e_i as

$$x_{id} = [y_{id}, \dot{y}_{id}, \dots, y_{id}^{(n_i-1)}]^T$$

$$e_i = x_i - x_{id} = [e_{i1}, e_{i2}, \dots, e_{in_i}]^T$$

and the filtered tracking error s_i as

$$s_i = \left(\frac{d}{dt} + \lambda_i \right)^{n_i-1} e_{i1}$$

$$= \sum_{k=0}^{n_i-1} C_{n_i-1}^k \lambda_i^{n_i-1-k} e_{i,k+1}$$

$$= \sum_{j=1}^{n_i-1} C_{n_i-1}^{j-1} \lambda_i^{n_i-j} e_{ij} + e_{in_i}$$

$$= \sum_{j=1}^{n_i-1} c_{ij} e_{ij} + e_{in_i} \quad (17)$$

where $c_{ij} = C_{n_i-1}^{j-1} \lambda_i^{n_i-j}$, $j = 1, \dots, n_i - 1$, $\lambda_i > 0$, $i = 1, \dots, m$ are positive constants, specified by designer.

Assumption 4: Smooth functions $b_i(\bar{x}_i)$ and their signs are unknown, and there exist constants b_{i0} and b_{i1} such that $0 < b_{i0} \leq |b_i(\bar{x}_i)| \leq b_{i1}$, $\forall \bar{x}_i \in R^{\bar{n}_i}$ with $\bar{n}_i = \sum_{j=1}^i n_j$, $i = 1, \dots, m$.

Assumption 5: The desired trajectory vectors are continuous and available, and $\bar{x}_{id} = [x_{id}^T, y_{id}^{(n_i)}]^T \in \Omega_{id} \subset R^{n_i+1}$ with Ω_{id} known compact set, $i = 1, \dots, m$.

B. Nussbaum Function Properties

In order to deal with the unknown control gain sign, the Nussbaum gain technique is employed in this paper. A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties:

- 1) $\lim_{s \rightarrow \infty} \sup \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty$
- 2) $\lim_{s \rightarrow \infty} \inf \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty$

Commonly used Nussbaum functions include: $\zeta^2 \cos(\zeta)$, $\zeta^2 \sin(\zeta)$, and $\exp(\zeta^2) \cos((\pi/2)\zeta)$ [11], [13], [22], [26]. For clarity, the even Nussbaum function $N(\zeta) = e^{\zeta^2} \cos((\pi/2)\zeta)$ is used throughout this paper.

Lemma 1 [12]: Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0, t_f]$ with $V(t) \geq 0$, $\forall t \in [0, t_f]$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \leq c_0 + \int_0^t (gN(\zeta) + 1) \dot{\zeta} d\tau \quad \forall t \in [0, t_f] \quad (18)$$

where g is a nonzero constant and c_0 represents some suitable constant, then $V(t)$, $\zeta(t)$, and $\int_0^t (gN(\zeta) + 1) \dot{\zeta} d\tau$ must be bounded on $[0, t_f]$.

Lemma 2 [13]: Let $V(\cdot)$, $\zeta(\cdot)$ be smooth functions defined on $[0, t_f]$ with $V(t) \geq 0$, $\forall t \in [0, t_f]$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$V(t) \leq c_0 + e^{-c_1 t} \int_0^t g(x(\tau)) N(\zeta) \dot{\zeta} e^{c_1 \tau} d\tau$$

$$+ e^{-c_1 t} \int_0^t \dot{\zeta} e^{c_1 \tau} d\tau, \quad \forall t \in [0, t_f] \quad (19)$$

where c_0 represents some suitable constant, c_1 is a positive constant, and $g(x(\tau))$ is a time-varying parameter, which takes values in the unknown closed intervals $I = [l^-, l^+]$, with $0 \notin I$, then $V(t)$, $\zeta(t)$, $\int_0^t g(x(\tau)) N(\zeta) \dot{\zeta} d\tau$ must be bounded on $[0, t_f]$.

Lemma 3 [26]: For any given positive constant $t_f > 0$, if the solution of the resulting closed-loop system is bounded on the interval $[0, t_f]$, then $t_f = \infty$.

In this paper, $\|\cdot\|$ denotes the 2-norm, $\|\cdot\|_F$ denotes the Frobenius norm, $\|A\|_1 = \sum_{k=1}^l |a_k|$ with $A = [a_1, \dots, a_l] \in R^l$, and $\lambda_{\min}(B)$ and $\lambda_{\max}(B)$ denote the smallest and largest eigenvalues of a square matrix B , respectively.

III. CONTROL SYSTEM DESIGN AND STABILITY ANALYSIS

A. Multilayer Neural Networks (MNNs)

NNs have been widely used in modeling and control of nonlinear systems because of their good capabilities of nonlinear function approximation, learning, and fault tolerance. In this paper, three-layer NNs will be used to approximate a continuous function $h(z) : R^p \rightarrow R$ as described by [27], [28]

$$h_{nn}(z, W, V) = W^T S(V^T \bar{z}) \quad (20)$$

where $z = [z_1, \dots, z_p]^T$, $\bar{z} = [z^T, 1]^T$, $V = [v_1, \dots, v_l] \in R^{(p+1) \times l}$ and $W = [w_1, \dots, w_l]^T \in R^l$ are the first-to-second layer and the second-to-third layer weights, respectively, $S(V^T \bar{z}) = [s(v_1^T \bar{z}), \dots, s(v_{l-1}^T \bar{z}), 1]^T$ with $s(z_\alpha) = 1/(1 + e^{-\gamma z_\alpha})$ and constant $\gamma > 0$, and the NN node number $l > 1$.

Let

$$h(z) = h_{nn}(z, W^*, V^*) + \varepsilon(z) \quad \forall z \in \Omega_z \subset R^p \quad (21)$$

where W^*, V^* are ideal NN weights, $\Omega_z \subset R^p$ is a compact set, and $\varepsilon(z)$ is the NN approximation error.

The ideal weights W^* and V^* are ‘‘artificial’’ quantity required for analytical purposes. W^* and V^* are defined as follows:

$$(W^*, V^*) = \arg \min_{(W, V)} \left[\sup_{z \in \Omega_z} |h_{nn}(z, W, V) - h(z)| \right]. \quad (22)$$

It is clear that W^* and V^* are usually unknown and need to be estimated in controller design. Let \hat{W} and \hat{V} be the estimates of W^* and V^* , respectively, and $(\hat{\cdot}) = (\cdot) - (*).$

Lemma 4 [28]: For NN (20), the NN estimation error can be expressed as

$$\hat{W}^T S(\hat{V}^T \bar{z}) - W^{*T} S(V^{*T} \bar{z}) = \tilde{W}^T (\hat{S} - \hat{S}' \hat{V}^T \bar{z}) + \hat{W}^T \hat{S}' \hat{V}^T \bar{z} + d_u \quad (23)$$

where $\hat{S} = S(\hat{V}^T \bar{z})$, $\hat{S}' = \text{diag}\{\hat{s}'_1, \dots, \hat{s}'_{l-1}, 0\}$ with $\hat{s}'_k = s'(v_k^T \bar{z}) = d[s(z_\alpha)]/dz_\alpha|_{z_\alpha=v_k^T \bar{z}}$, $k = 1, \dots, l-1$, and the residual term d_u is bounded by

$$\|d_u\| \leq \|V^*\|_F \|\tilde{z} \hat{W}^T \hat{S}'\|_F + \|W^*\| \|\hat{S}' \hat{V}^T \bar{z}\| + \|W^*\|_1. \quad (24)$$

According to (21) and (23), we obtain

$$h(z) = \hat{W}^T S(\hat{V}^T \bar{z}) - \tilde{W}^T (\hat{S} - \hat{S}' \hat{V}^T \bar{z}) - \hat{W}^T \hat{S}' \hat{V}^T \bar{z} - d_u + \varepsilon(z). \quad (25)$$

B. Adaptive NN Control for SISO System ($m = 1$)

To illustrate the design methodology clearly, we first consider the SISO system ($m = 1$).

From (1), (9), and (16), we obtain

$$\dot{s}_1 = f_1(x_1) + \nu_1 + b_1(x_1) K_1^T(v_1(t)) \Phi_1 \times (v_1(t)) v_1(t) + b_1(x_1) d_1(v_1(t)) \quad (26)$$

where $\nu_1 = \sum_{j=1}^{n_1-1} c_{1j} e_{1,j+1} - y_{1d}^{(n_1)}$.

To avoid control singularity, we employ integral Lyapunov function [4]. In this paper, we define a smooth scalar function as follows:

$$V_{s_1} = \int_0^{s_1} \frac{\sigma}{|b_1(\bar{x}_1^+ + \sigma + \beta_1)|} d\sigma \quad (27)$$

where $\beta_1 = y_{1d}^{(n_1-1)} - \sum_{j=1}^{n_1-1} c_{1j} e_{1j}$ and $\bar{x}_1^+ = [x_{11}, \dots, x_{1,n_1-1}]^T$.

By second mean value theorem for integrals, V_{s_1} can be rewritten as

$$V_{s_1} = \frac{1}{|b_1(\bar{x}_1^+ + \lambda_{s_1} s_1 + \beta_1)|} \int_0^{s_1} \sigma d\sigma = \frac{s_1^2}{2|b_1(\bar{x}_1^+ + \lambda_{s_1} s_1 + \beta_1)|} \quad (28)$$

with $\lambda_{s_1} \in (0, 1)$. Because $0 < b_{10} \leq |b_1(x_1)|$, it is shown that V_{s_1} is positive definite with respect to s_1 .

Differentiating V_{s_1} with respect to time t , we obtain

$$\dot{V}_{s_1} = \frac{s_1}{|b_1(x_1)|} \dot{s}_1 + \int_0^{s_1} \sigma \left[\sum_{k=1}^{n_1-1} \frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial x_{1k}} x_{1,k+1} + \frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial \beta_1} \dot{\beta}_1 \right] d\sigma. \quad (29)$$

Because $\dot{\beta}_1 = -\nu_1$ and $\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)| / \partial \sigma = \partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)| / \partial \beta_1$, it is shown that

$$\int_0^{s_1} \sigma \frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial \beta_1} \dot{\beta}_1 d\sigma = \int_0^{s_1} \sigma \left[\frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial \sigma} \right] (-\nu_1) d\sigma = -\frac{\nu_1 s_1}{|b_1(x_1)|} + \int_0^{s_1} \frac{\nu_1}{|b_1(\bar{x}_1^+ + \sigma + \beta_1)|} d\sigma. \quad (30)$$

Substituting (26) and (30) into (29), and applying (16), we obtain

$$\dot{V}_{s_1} = \frac{s_1}{|b_1(x_1)|} \dot{s}_1 + \int_0^{s_1} \sigma \left[\sum_{k=1}^{n_1-1} \frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial x_{1k}} x_{1,k+1} \right] d\sigma - \frac{\nu_1 s_1}{|b_1(x_1)|} + \nu_1 \int_0^{s_1} |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)| d\sigma \leq s_1 g_1(t) v_1(t) + s_1 h_1(z_1) + |s_1| p_1^* \quad (31)$$

where

$$g_1(t) = \frac{b_1(x_1)}{|b_1(x_1)|} K_1^T(v_1(t)) \Phi_1(v_1(t)) \quad (32)$$

$$h_1(z_1) = \frac{f_1(x_1)}{|b_1(x_1)|} + \frac{1}{s_1} \int_0^{s_1} \left[\sigma \sum_{k=1}^{n_1-1} \frac{\partial |b_1^{-1}(\bar{x}_1^+ + \sigma + \beta_1)|}{\partial x_{1k}} x_{1,k+1} + \frac{\nu_1}{|b_1(\bar{x}_1^+ + \sigma + \beta_1)|} \right] d\sigma = \frac{f_1(x_1)}{|b_1(x_1)|}$$

$$\begin{aligned}
 & + \int_0^1 \left[\theta s_1 \sum_{k=1}^{n_1-1} \frac{\partial |b_1^{-1}(\bar{x}_1^+, \theta s_1 + \beta_1)|}{\partial x_{1k}} x_{1,k+1} \right. \\
 & \quad \left. + \frac{\nu_1}{|b_1(\bar{x}_1^+, \theta s_1 + \beta_1)|} \right] d\theta \quad (33)
 \end{aligned}$$

with $z_1 = [x_1^T, s_1, \nu_1, \beta_1]^T \in R^{p_1}$ and $p_1 = n_1 + 3$.

Define a compact set

$$\Omega_{z_1} = \left\{ [x_1^T, s_1, \nu_1, \beta_1]^T \mid x_1 \in \Omega_1, \bar{x}_{1d} \in \Omega_{1d} \right\} \quad (34)$$

where $\Omega_1 \subset R^{n_1}$ is a sufficiently large compact set satisfying $\Omega_1 \supset \Omega_{10}$, which will be defined later in step 2 in Theorem 1.

Let $\hat{W}_1^T S_1(\hat{V}_1^T \bar{z}_1)$ be the approximation of the three-layer NNs, which are discussed in Section III-A, on the compact Ω_{z_1} to $h_1(z_1)$, then we have

$$\begin{aligned}
 h_1(z_1) &= \hat{W}_1^T S_1(\hat{V}_1^T \bar{z}_1) - \tilde{W}_1^T (\hat{S}_1 - \hat{S}'_1 \hat{V}_1^T \bar{z}_1) \\
 &\quad - \hat{W}_1^T \hat{S}'_1 \hat{V}_1^T \bar{z}_1 - d_{u1} + \varepsilon_1(z_1) \quad (35)
 \end{aligned}$$

where $z_1 = [z_{11}, \dots, z_{1p_1}]^T = [x_1^T, s_1, \nu_1, \beta_1]^T$, $\bar{z}_1 = [z_1^T, 1]^T$, $\hat{V}_1 = [\hat{v}_{11}, \dots, \hat{v}_{1l_1}] \in R^{(p_1+1) \times l_1}$ and $\hat{W}_1 = [\hat{w}_{11}, \dots, \hat{w}_{1l_1}]^T \in R^{l_1}$ denote the estimates of V_1^* and W_1^* , respectively, W_1^* and V_1^* are ideal constant weights, $\hat{S}_1 = S_1(\hat{V}_1^T \bar{z}_1) = [s(\hat{v}_{11}^T \bar{z}_1), \dots, s(\hat{v}_{1l_1}^T \bar{z}_1), 1]^T$ with $s(z_\alpha) = 1/(1 + e^{-\gamma_{10} z_\alpha})$, and constant $\gamma_{10} > 0$, $\hat{S}'_1 = \text{diag}\{\hat{s}'_{11}, \dots, \hat{s}'_{1, l_1-1}, 0\}$ with $\hat{s}'_{1k} = s'(\hat{v}_{1k}^T \bar{z}_1) = d[s(z_\alpha)]/dz_\alpha|_{z_\alpha=\hat{v}_{1k}^T \bar{z}_1}$, $k = 1, \dots, l_1 - 1$, the NN node number $l_1 > 1$, the residual term d_{u1} is bounded by

$$|d_{u1}| \leq \|V_1^*\|_F \|\bar{z}_1 \hat{W}_1^T \hat{S}'_1\|_F + \|W_1^*\| \|\hat{S}'_1 \hat{V}_1^T \bar{z}_1\| + \|W_1^*\|_1 \quad (36)$$

and the approximation error $\varepsilon_1(z_1)$ in (35) satisfies $|\varepsilon_1(z_1)| \leq \varepsilon_1^*$, $\forall z_1 \in \Omega_{z_1}$ with constant $\varepsilon_1^* > 0$.

Consider the following control law:

$$\begin{aligned}
 v_1(t) &= N(\zeta_1) \left[k_{10} s_1 + \hat{W}_1^T S_1(\hat{V}_1^T \bar{z}_1) \right. \\
 &\quad \left. + \hat{\delta}_1 \xi_1(z_1) \tanh(s_1 \xi_1(z_1)/\rho_1) \right] \quad (37)
 \end{aligned}$$

$$\begin{aligned}
 \dot{\zeta}_1 &= k_{10} s_1^2 + \hat{W}_1^T S_1(\hat{V}_1^T \bar{z}_1) s_1 \\
 &\quad + \hat{\delta}_1 \xi_1(z_1) s_1 \tanh(s_1 \xi_1(z_1)/\rho_1) \quad (38)
 \end{aligned}$$

where $N(\zeta_1) = e^{\zeta_1^2} \cos((\pi/2)\zeta_1)$, k_{10} , and ρ_1 are positive constants, $\hat{\delta}_1$ is the estimate of δ_1^* with $\delta_1^* = \max\{\|V_1^*\|_F, \|W_1^*\|, \|W_1^*\|_1 + \varepsilon_1^* + p_1^*\}$ at time t , and

$$\xi_1(z_1) = \|\bar{z}_1 \hat{W}_1^T \hat{S}'_1\|_F + \|\hat{S}'_1 \hat{V}_1^T \bar{z}_1\| + 1. \quad (39)$$

The adaptive laws are employed as follows:

$$\dot{\hat{W}}_1 = \Gamma_{w1} \left[(\hat{S}_1 - \hat{S}'_1 \hat{V}_1^T \bar{z}_1) s_1 - \sigma_{w1} \hat{W}_1 \right] \quad (40)$$

$$\dot{\hat{V}}_1 = \Gamma_{v1} \left[\bar{z}_1 \hat{W}_1^T \hat{S}'_1 s_1 - \sigma_{v1} \hat{V}_1 \right] \quad (41)$$

$$\dot{\hat{\delta}}_1 = \eta_1 \left[s_1 \xi_1(z_1) \tanh(s_1 \xi_1(z_1)/\rho_1) - \sigma_1 \hat{\delta}_1 \right] \quad (42)$$

where $\Gamma_{w1} > 0$, $\Gamma_{v1} > 0$, σ_{w1} , σ_{v1} , η_1 , and σ_1 are strictly positive constants.

Theorem 1: Consider the closed-loop system consisting of the plant (1), the adaptive control given by (37) and (40)–(42). Under Assumptions 1–5, for bounded initial conditions, the overall closed-loop neural control system is semiglobally stable in the sense that all of the signals in the closed-loop system are bounded, the parameter estimates

$$\begin{aligned}
 & (\hat{W}_1, \hat{V}_1, \hat{\delta}_1) \in \Omega_{w1} \\
 & = \left\{ (\hat{W}_1, \hat{V}_1, \hat{\delta}_1) \mid \|\tilde{W}_1\|^2 \leq 2\mu_1 [\lambda_{\min}(\Gamma_{w1}^{-1})]^{-1}, \right. \\
 & \quad \left. \|\tilde{V}_1\|_F^2 \leq \frac{2\mu_1}{\lambda_{\min}(\Gamma_{v1}^{-1})}, |\tilde{\delta}_1|^2 \leq 2\eta_1 \mu_1 \right\} \quad (43)
 \end{aligned}$$

and $\forall x_1(0) \in \Omega_{10}$, the state vector

$$\begin{aligned}
 x_1 \in \Omega_{1c} &= \left\{ x_1 \mid \|x_1 - x_{1d}\| \leq c_{10} (1 + \|\Lambda_1\|) \|\omega_1(0)\| \right. \\
 &\quad \left. + \left[1 + \frac{(1 + \|\Lambda_1\|) c_{10}}{\lambda_1} \right] \sqrt{2b_{11}\mu_1}, \bar{x}_{1d} \in \Omega_{1d} \right\} \\
 &\subset \Omega_1 \quad (44)
 \end{aligned}$$

where $\mu_1 > 0$ and $c_{10} > 0$, and the compact set Ω_{10} and the vector Λ_1 will be defined later in the proof.

Proof: The proof includes two steps. We will first assume that $x_1(t) \in \Omega_1$, $\forall t \geq 0$, on which NN approximation (35) is valid, and construct stable adaptive NN control over Ω_1 . Then, we will show that there exists nonempty initial set Ω_{10} such that the state $x_1(t)$ indeed remains in the compact set Ω_1 for all $t \geq 0$ if initial state $x_1(0)$ initiates from Ω_{10} .

We will first assume that $x_1 \in \Omega_1$ holds for all time, and find the upper bounds of system states. Later, for the appropriate initial condition $x_1(0)$ and the adaptive controller parameters, we prove that state $x_1(t)$ indeed remains in the compact set Ω_1 for all $t \geq 0$.

Step 1: Suppose that $x_1(t) \in \Omega_1$, $\forall t \geq 0$, then NN approximation (35) is valid. Consider the Lyapunov function candidate $V_1(t) = V_{s_1}(t) + \frac{1}{2} \tilde{W}_1^T \Gamma_{w1}^{-1} \tilde{W}_1 + \frac{1}{2} \text{tr} \left\{ \tilde{V}_1^T \Gamma_{v1}^{-1} \tilde{V}_1 \right\} + \frac{1}{2\eta_1} \tilde{\delta}_1^2$. (45)

Differentiating $V_1(t)$ with respect to time t leads to

$$\dot{V}_1(t) = \dot{V}_{s_1}(t) + \tilde{W}_1^T \Gamma_{w1}^{-1} \dot{\tilde{W}}_1 + \text{tr} \left\{ \tilde{V}_1^T \Gamma_{v1}^{-1} \dot{\tilde{V}}_1 \right\} + \frac{1}{\eta_1} \dot{\tilde{\delta}}_1 \dot{\delta}_1. \quad (46)$$

Substituting inequality (31) into (46), noting (35) and (36), and using control law (37) and (38), it follows that

$$\begin{aligned}
 \dot{V}_1(t) &\leq g_1(t) N(\zeta_1) \dot{\zeta}_1 + \dot{\zeta}_1 - k_{10} s_1^2 - \hat{W}_1^T S_1(\hat{V}_1^T \bar{z}_1) s_1 \\
 &\quad - \hat{\delta}_1 \xi_1(z_1) s_1 \tanh(s_1 \xi_1(z_1)/\rho_1) \\
 &\quad + \left[\hat{W}_1^T S(\hat{V}_1^T \bar{z}_1) - \tilde{W}_1^T (\hat{S}_1 - \hat{S}'_1 \hat{V}_1^T \bar{z}_1) \right. \\
 &\quad \quad \left. - \hat{W}_1^T \hat{S}'_1 \hat{V}_1^T \bar{z}_1 - d_{u1} + \varepsilon_1(z_1) \right] s_1 \\
 &\quad + |s_1| p_1^* + \tilde{W}_1^T \Gamma_{w1}^{-1} \dot{\tilde{W}}_1 + \text{tr} \left\{ \tilde{V}_1^T \Gamma_{v1}^{-1} \dot{\tilde{V}}_1 \right\} \\
 &\quad + \frac{1}{\eta_1} \dot{\tilde{\delta}}_1 \dot{\delta}_1. \quad (47)
 \end{aligned}$$

Using adaptive tuning laws (40)–(42), and the inequality $0 \leq |r| - r \tanh(r/\varepsilon_0) \leq 0.2785\varepsilon_0$, for $\varepsilon_0 > 0$, $r \in R$, and the fact that $\dot{W}_1^T \hat{S}_1^T \hat{V}_1^T \hat{z}_1 = \text{tr}\{\hat{V}_1^T \hat{z}_1 \dot{W}_1^T \hat{S}_1^T\}$, we obtain

$$\begin{aligned} \dot{V}_1(t) &\leq g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - k_{10}s_1^2 + \delta_1^*\xi_1(z_1)|s_1| \\ &\quad - \delta_1^*\xi_1(z_1)s_1 \tanh(s_1\xi_1(z_1)/\rho_1) - \sigma_{w1}\tilde{W}_1^T\hat{W}_1 \\ &\quad - \sigma_{v1}\text{tr}\left\{\tilde{V}_1^T\hat{V}_1\right\} - \sigma_1\tilde{\delta}_1\hat{\delta}_1 \\ &\leq g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - k_{10}s_1^2 + 0.2785\delta_1^*\rho_1 \\ &\quad - \sigma_{w1}\tilde{W}_1^T\hat{W}_1 - \sigma_{v1}\text{tr}\left\{\tilde{V}_1^T\hat{V}_1\right\} - \sigma_1\tilde{\delta}_1\hat{\delta}_1. \end{aligned} \quad (48)$$

By completion of squares, the following inequalities hold:

$$-\sigma_{w1}\tilde{W}_1^T\hat{W}_1 \leq -\frac{\sigma_{w1}\|\tilde{W}_1\|^2}{2} + \frac{\sigma_{w1}\|W_1^*\|^2}{2} \quad (49)$$

$$-\sigma_{v1}\text{tr}\left\{\tilde{V}_1^T\hat{V}_1\right\} \leq -\frac{\sigma_{v1}\|\tilde{V}_1\|_F^2}{2} + \frac{\sigma_{v1}\|V_1^*\|_F^2}{2} \quad (50)$$

$$-\sigma_1\tilde{\delta}_1\hat{\delta}_1 \leq -\frac{\sigma_1\tilde{\delta}_1^2}{2} + \frac{\sigma_1\delta_1^{*2}}{2}. \quad (51)$$

We obtain

$$\begin{aligned} \dot{V}_1(t) &\leq -k_{10}s_1^2 + g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + 0.2785\delta_1^*\rho_1 \\ &\quad - \frac{\sigma_{w1}\|\tilde{W}_1\|^2}{2} - \frac{\sigma_{v1}\|\tilde{V}_1\|_F^2}{2} - \frac{\sigma_1\tilde{\delta}_1^2}{2} + \frac{\sigma_{w1}\|W_1^*\|^2}{2} \\ &\quad + \frac{\sigma_{v1}\|V_1^*\|_F^2}{2} + \frac{\sigma_1\delta_1^{*2}}{2}. \end{aligned} \quad (52)$$

Define the following constants:

$$\lambda_{10} = \min\{2k_{10}b_{10}, \sigma_{w1}/\lambda_{\max}(\Gamma_{w1}^{-1}), \sigma_{v1}/\lambda_{\max}(\Gamma_{v1}^{-1}), \sigma_1\eta_1\} \quad (53)$$

$$\mu_{10} = 0.2785\delta_1^*\rho_1 + \frac{\sigma_{w1}\|W_1^*\|^2}{2} + \frac{\sigma_{v1}\|V_1^*\|_F^2}{2} + \frac{\sigma_1\delta_1^{*2}}{2}. \quad (54)$$

Thus, we have

$$\dot{V}_1(t) \leq -\lambda_{10}V_1(t) + \mu_{10} + g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1. \quad (55)$$

Multiplying inequality (55) by $e^{\lambda_{10}t}$ yields

$$\frac{d}{dt}(V_1(t)e^{\lambda_{10}t}) \leq e^{\lambda_{10}t}\mu_{10} + e^{\lambda_{10}t}(g_1(t)N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1). \quad (56)$$

Integrating (56) over $[0, t]$, we have

$$\begin{aligned} 0 \leq V_1(t) &\leq \frac{\mu_{10}}{\lambda_{10}} + \left[V_1(0) - \frac{\mu_{10}}{\lambda_{10}}\right]e^{-\lambda_{10}t} \\ &\quad + e^{-\lambda_{10}t} \int_0^t (g_1(\tau)N(\zeta_1) + 1)\dot{\zeta}_1 e^{\lambda_{10}\tau} d\tau \\ &\leq \frac{\mu_{10}}{\lambda_{10}} + V_1(0) \\ &\quad + e^{-\lambda_{10}t} \int_0^t (g_1(\tau)N(\zeta_1) + 1)\dot{\zeta}_1 e^{\lambda_{10}\tau} d\tau. \end{aligned} \quad (57)$$

From (10)–(15) and Assumption 3, we know that $g_1(t) \in [\min\{k_{1l0}, k_{1r0}\}, k_{1l1} + k_{1r1}] \subset (0, +\infty)$ or $g_1(t) \in$

$[-(k_{1l1} + k_{1r1}), -\min\{k_{1l0}, k_{1r0}\}] \subset (-\infty, 0)$. According to Lemma 2, we have that $V_1(t), \zeta_1(t), \int_0^t g_1(\tau)N(\zeta_1)\dot{\zeta}_1 d\tau$ are bounded in $[0, t_f]$. According to Lemma 3, we know that the above conclusion is true for $t_f = +\infty$. Therefore, $\tilde{\delta}_1, \|\tilde{W}_1\|$, and $\|\tilde{V}_1\|_F \in L_\infty$. Let C_{ζ_1} be the upper bound of $e^{-\lambda_{10}t} \int_0^t (g_1(\tau)N(\zeta_1) + 1)\dot{\zeta}_1 e^{\lambda_{10}\tau} d\tau$ in $[0, \infty)$, and

$$\mu_1 = \frac{\mu_{10}}{\lambda_{10}} + V_1(0) + C_{\zeta_1} \quad (58)$$

then $s_1^2 \leq 2b_{11}V_1(t) \leq 2b_{11}\mu_1$. Similarly, $\|\tilde{W}_1\|^2 \leq 2\mu_1/\lambda_{\min}(\Gamma_{w1}^{-1})$, $\|\tilde{V}_1\|_F^2 \leq 2\mu_1/\lambda_{\min}(\Gamma_{v1}^{-1})$, and $|\tilde{\delta}_1|^2 \leq 2\eta_1\mu_1$.

Define $\omega_1 = [e_{11}, \dots, e_{1, n_1-1}]^T \in R^{n_1-1}$. From equation (17), we know that 1) there is a state-space representation for mapping $s_1 = [\Lambda_1^T 1]e_1$, i.e., $\dot{\omega}_1 = A_{s_1}\omega_1 + b_{s_1}s_1$ with $\Lambda_1 = [\lambda_{11}, \dots, \lambda_{1, n_1-1}]^T$, $b_{s_1} = [0, \dots, 0, 1]^T$, A_{s_1} being a stable matrix; 2) there is a positive constant c_{10} such that $\|e^{A_{s_1}t}\| \leq c_{10}e^{-\lambda_1 t}$; and 3) the solution for ω_1 is

$$\omega_1(t) = e^{A_{s_1}t}\omega_1(0) + \int_0^t e^{A_{s_1}(t-\tau)}b_{s_1}s_1(\tau)d\tau.$$

Accordingly, it follows that

$$\|\omega_1(t)\| \leq c_{10}\|\omega_1(0)\|e^{-\lambda_1 t} + c_{10} \int_0^t e^{-\lambda_1(t-\tau)}|s_1(\tau)|d\tau.$$

Therefore, we have

$$\|\omega_1(t)\| \leq c_{10}\|\omega_1(0)\| + \frac{c_{10}\sqrt{2b_{11}\mu_1}}{\lambda_1}. \quad (59)$$

Noting $s_1 = \Lambda_1^T\omega_1 + e_{1n_1}$ and $e_1 = [\omega_1^T, e_{1n_1}]^T$, we obtain

$$\|e_1\| \leq \|\omega_1\| + |e_{1n_1}| \leq (1 + \|\Lambda_1\|)\|\omega_1\| + |s_1|.$$

Substituting inequality (59) into the above inequality leads to

$$\|e_1\| \leq c_{10}(1 + \|\Lambda_1\|)\|\omega_1(0)\| + \left[1 + \frac{(1 + \|\Lambda_1\|)c_{10}}{\lambda_1}\right]\sqrt{2b_{11}\mu_1}. \quad (60)$$

Since $c_{10}, \|\Lambda_1\|$, and λ_1 are positive constants, and $\omega_1(0)$ and $s_1(0)$ depend on $x_1(0) - x_{1d}(0)$, we conclude that there exists a positive constant $R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0))$ depending on $c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0)$, and $\tilde{\delta}_1(0)$ such that

$$\|e_1\| \leq R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)) \quad \forall t \geq 0 \quad (61)$$

with $c_1 = \mu_{10}/\lambda_{10}$, and

$$R_1(c_1, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)) = c_{10}(1 + \|\Lambda_1\|)\|\omega_1(0)\| + \left[1 + \frac{(1 + \|\Lambda_1\|)c_{10}}{\lambda_1}\right]\sqrt{2b_{11}\mu_1}. \quad (62)$$

Noting $x_1 = e_1 + x_{1d}$ and Assumption 5, we obtain

$$\|x_1\| \leq \|e_1\| + \|x_{1d}\| \leq c_{10}(1 + \|\Lambda_1\|)\|\omega_1(0)\| + \left[1 + \frac{(1 + \|\Lambda_1\|)c_{10}}{\lambda_1}\right]\sqrt{2b_{11}\mu_1} + \|x_{1d}\| \in L_\infty.$$

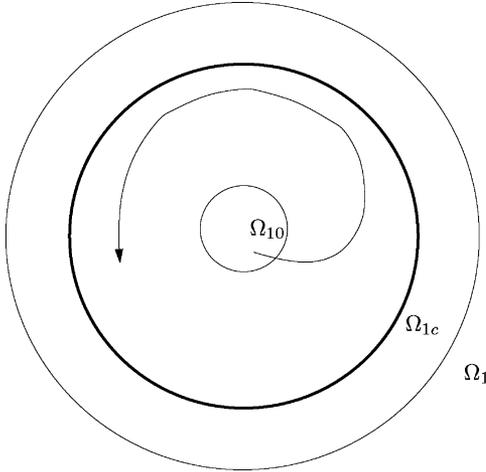


Fig. 2. Compact sets in Theorem 1.

Therefore, we can conclude that all the closed-loop signals are semiglobally uniformly ultimately bounded for bounded initial conditions.

Step 2: In the following, we will find the conditions such that $x_1(t) \in \Omega_1, \forall t \geq 0$. First, define a set

$$\Omega_{10} = \{x_1(0) | \{x_1(t) | \|x_1(t) - x_{1d}(t)\| < R_1(0, x_1(0), 0, 0, 0), \forall t \geq 0\} \subset \Omega_1, x_1(0) \in \Omega_1, \bar{x}_{1d} \in \Omega_{1d}\}. \quad (63)$$

It is easy to see that for all $x_1(0) \in \Omega_{10}$ and $\bar{x}_{1d} \in \Omega_{1d}$, we have $x_1(t) \in \Omega_1, \forall t \geq 0$. Then, for the system with $x_1(0) \in \Omega_{10}$, bounded $\tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)$, and $\bar{x}_{1d} \in \Omega_{1d}$, the following constant c_1^* can be determined by:

$$\begin{aligned} c_1^* &= \sup_{c \in \mathbb{R}^+} \{c | \{x_1(t) | \|x_1(t) - x_{1d}(t)\| \\ &\leq R_1(c, x_1(0), \tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)), \forall t \geq 0\} \\ &\subset \Omega_1, x_1(0) \in \Omega_1, \bar{x}_{1d} \in \Omega_{1d}\}. \end{aligned} \quad (64)$$

From (53) and (54), we know that if the adaptive control parameters $\sigma_{w1}, \sigma_{v1}, \sigma_1$, and ρ_1 are chosen to be sufficiently small and $k_{10}, \eta_1, \lambda_{\min}(\Gamma_{w1})$, and $\lambda_{\min}(\Gamma_{v1})$ are taken to be sufficiently large, then the constant $c_1 = \mu_{10}/\lambda_{10}$ can be made arbitrary small. Therefore, for the initial condition $x_1(0) \in \Omega_{10}$, bounded $\tilde{W}_1(0), \tilde{V}_1(0), \tilde{\delta}_1(0)$, and $x_{1d} \in \Omega_{1d}$, if the adaptive control parameters are appropriately chosen such that $c_1 = \mu_{10}/\lambda_{10} \leq c_1^*$, then system state $x_1(t)$ indeed stays in Ω_1 for all time t .

Through the process of the proof, it is clear that there is a nonempty initial compact set Ω_{10} ; as long as initial state $x_1(0)$ starts from Ω_{10} , the state $x_1(t)$ will never escape out of the conservative compact set Ω_{1c} , belonging to the chosen compact set Ω_1 for all time t . Because NN approximation is only valid on a compact set, we have to present the idea in the above manner, and at the same time avoid the so-called circular argument as commonly understood in the classical-model-based control. To help understand the above proof, the defined three compact sets in Theorem 1 are shown in Fig. 2 as discussed in [29].

Remark 2: Because the output of an unknown dead zone is not available, controller design based on the inverse of a dead zone usually needs to estimate some unknown parameters

related to the dead zone [14], [19], [23], [30]. However, for controller design based on (9), there is no need to estimate the parameters related to dead zones, and can avoid the need for parameter bounds of dead zones (for example, the parameters b_l and b_r of the dead zone need to be known in [19, eq. (22)]; the parameters u_{i-}, u_{i+} of the dead zones also need to be known in [21, eq. (14)]). In addition, since b_{ir} and b_{il} are unknown, $\varphi_{ir}(v_i(t))$ and $\varphi_{il}(v_i(t))$ are unknown as well, i.e., both $\Phi_i(v_i(t))$ and $K_i(v_i(t))$ cannot be computable. However, the particular description (9) makes the control system design possible without necessarily constructing a dead-zone inverse by utilizing Lemmas 1–3.

C. Adaptive NN Control for MIMO System ($m \geq 2$)

In this section, the design in Section III-B is extended to MIMO system (1), which contains m interconnected subsystems. For the i th subsystem

$$\begin{cases} \dot{x}_{ij} = x_{i,j+1}, j = 1, \dots, n_i - 1 \\ \dot{x}_{in_i} = f_i(x, u_1, \dots, u_{i-1}) + b_i(x_1, \dots, x_i)u_i, i = 2, \dots, m \\ y_i = x_{i1} \end{cases} \quad (65)$$

the filtering tracking error s_i is determined by (17). From (9), (16), and (65), we obtain

$$\begin{aligned} \dot{s}_i &= f_i(x, u_1, \dots, u_{i-1}) + \nu_i + b_i(\bar{x}_i)K_i^T(v_i(t))\Phi_i \\ &\quad \times (v_i(t))v_i(t) + b_i(\bar{x}_i)d_i(v_i(t)) \end{aligned} \quad (66)$$

where $\nu_i = \sum_{j=1}^{n_i-1} c_{ij}e_{i,j+1} - y_{id}^{(n_i)}$.

Define a smooth scalar function as follows:

$$V_{s_i} = \int_0^{s_i} \frac{\sigma}{|b_i(\bar{x}_i^+, \sigma + \beta_i)|} d\sigma \quad (67)$$

where $\beta_i = y_{id}^{(n_i-1)} - \sum_{j=1}^{n_i-1} c_{ij}e_{ij}$ and $\bar{x}_i^+ = [x_1^T, \dots, x_{i-1}^T, x_{i1}, \dots, x_{i, n_i-1}]^T$.

By second mean value theorem for integrals, V_{s_i} can be rewritten as

$$\begin{aligned} V_{s_i} &= \frac{1}{|b_i(\bar{x}_i^+, \lambda_{s_i} s_i + \beta_i)|} \int_0^{s_i} \sigma d\sigma \\ &= \frac{s_i^2}{2|b_i(\bar{x}_i^+, \lambda_{s_i} s_i + \beta_i)|} \end{aligned} \quad (68)$$

with $\lambda_{s_i} \in (0, 1)$. Because $0 < b_{i0} \leq |b_i(\bar{x}_i)|$, it is shown that V_{s_i} is positive definite with respect to s_i .

Differentiating V_{s_i} with respect to time t and applying (16) and (66), we obtain

$$\begin{aligned} \dot{V}_{s_i} &= \frac{s_i}{|b_i(\bar{x}_i)|} \dot{s}_i + \int_0^{s_i} \sigma \left[\sum_{j=1}^{i-1} \sum_{k=1}^{n_j} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial x_{jk}} x_{j,k+1} \right. \\ &\quad \left. + \sum_{k=1}^{n_i-1} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial x_{ik}} x_{i,k+1} \right. \\ &\quad \left. + \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial \beta_i} \dot{\beta}_i \right] d\sigma \\ &= \frac{s_i}{|b_i(\bar{x}_i)|} \dot{s}_i \end{aligned}$$

$$\begin{aligned}
& + \int_0^{s_i} \sigma \left[\sum_{j=1}^{i-1} \sum_{k=1}^{n_j} \frac{\partial |b_i^{-1}(\bar{x}_i^+ \sigma + \beta_i)|}{\partial x_{jk}} x_{j,k+1} \right. \\
& \quad \left. + \sum_{k=1}^{n_i-1} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial x_{ik}} x_{i,k+1} \right] d\sigma \\
& - \frac{\nu_i s_i}{|b_i(\bar{x}_i)|} + \nu_i \int_0^{s_i} |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)| d\sigma \\
& \leq s_i g_i(t) \nu_i(t) + s_i h_i(z_i) + |s_i| p_i^* \tag{69}
\end{aligned}$$

where

$$g_i(t) = \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} K_i^T(\nu_i(t)) \Phi_i(\nu_i(t)) \tag{70}$$

$$\begin{aligned}
h_i(z_i) &= \frac{f_i(x, u_1, \dots, u_{i-1})}{|b_i(\bar{x}_i)|} \\
& + \frac{1}{s_i} \int_0^{s_i} \left\{ \sigma \left[\sum_{j=1}^{i-1} \sum_{k=1}^{n_j} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial x_{jk}} x_{j,k+1} \right. \right. \\
& \quad \left. \left. + \sum_{k=1}^{n_i-1} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)|}{\partial x_{ik}} x_{i,k+1} \right] \right. \\
& \quad \left. + \nu_i |b_i^{-1}(\bar{x}_i^+, \sigma + \beta_i)| \right\} d\sigma \\
& = \frac{f_i(x, u_1, \dots, u_{i-1})}{|b_i(\bar{x}_i)|} \\
& + \int_0^1 \left\{ \theta s_i \left[\sum_{j=1}^{i-1} \sum_{k=1}^{n_j} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \theta s_i + \beta_i)|}{\partial x_{jk}} x_{j,k+1} \right. \right. \\
& \quad \left. \left. + \sum_{k=1}^{n_i-1} \frac{\partial |b_i^{-1}(\bar{x}_i^+, \theta s_i + \beta_i)|}{\partial x_{ik}} x_{i,k+1} \right] \right. \\
& \quad \left. + \nu_i |b_i^{-1}(\bar{x}_i^+, \theta s_i + \beta_i)| \right\} d\theta \tag{71}
\end{aligned}$$

$$\begin{aligned}
z_i &= [z_{i1}, z_{i2}, \dots, z_{ip_i}]^T \\
&= [x^T, s_i, \nu_i, \beta_i, v_1, \dots, v_{i-1}]^T, \quad p_i = n + i + 2 \\
x_{j,n_j+1} &= \dot{x}_{jn_j}, \quad j = 1, \dots, i-1, i = 2, \dots, m. \tag{72}
\end{aligned}$$

Remark 3: Since $x_{j,n_j+1} = \dot{x}_{jn_j} = f_j(x, u_1, \dots, u_{j-1}) + b_j(\bar{x}_j) D_j(v_j)$, $j = 1, \dots, i-1$, we may conclude that the right-hand side of (71) is a function of variable $z_i = [x^T, s_i, \nu_i, \beta_i, v_1, \dots, v_{i-1}]^T$.

Define the compact sets Ω_{z_1} and Ω_{z_i} as follows:

$$\Omega_{z_1} = \{[x^T, s_1, \nu_1, \beta_1]^T | x_j \in \Omega_j, j = 1, \dots, m, \bar{x}_{1d} \in \Omega_{1d}\} \tag{73}$$

$$\begin{aligned}
\Omega_{z_i} &= \{[x^T, s_i, \nu_i, \beta_i, v_1, \dots, v_{i-1}]^T | x_j \in \Omega_j, j = 1, \dots, m, \\
& \quad \bar{x}_{kd} \in \Omega_{kd}, k = 1, \dots, i, \\
& \quad (\hat{W}_l, \hat{V}_l, \hat{\delta}_l) \in \Omega_{wl}, l = 1, \dots, i-1\} \tag{74}
\end{aligned}$$

where $\Omega_j \subset R^{n_j}$ is a sufficiently large compact set satisfying $\Omega_j \supset \Omega_{j0}$, $j = 1, \dots, m$; Ω_{j0}, Ω_{wl} will be defined later in Theorem 2, $l = 1, \dots, i-1, i = 2, \dots, m$.

Let $\hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i)$ be the approximation of the three-layer NNs, which are discussed in Section III-A, on the compact Ω_{z_i} to $h_i(z_i)$; then we have

$$\begin{aligned}
h_i(z_i) &= \hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i) - \tilde{W}_i^T (\hat{S}_i - \hat{S}_i' \hat{V}_i^T \bar{z}_i) \\
& \quad - \hat{W}_i^T \hat{S}_i' \hat{V}_i^T \bar{z}_i - d_{ui} + \varepsilon_i(z_i) \tag{75}
\end{aligned}$$

where $z_i = [z_{i1}, \dots, z_{ip_i}]^T$ and $\bar{z}_i = [z_i^T, 1]^T$, $\hat{V}_i = [\hat{v}_{i1}, \dots, \hat{v}_{il_i}] \in R^{(p_i+1) \times l_i}$ and $\hat{W}_i = [\hat{w}_{i1}, \dots, \hat{w}_{il_i}]^T \in R^{l_i}$ denote the estimates of V_i^* and W_i^* , respectively, W_i^* and V_i^* are ideal constant weights, $\hat{S}_i = S_i(\hat{V}_i^T \bar{z}_i) = [s(\hat{v}_{i1}^T \bar{z}_i), \dots, s(\hat{v}_{i,l_i-1}^T \bar{z}_i), 1]^T$ with $s(z_\alpha) = 1/(1 + e^{-\gamma_{i0} z_\alpha})$, and constant $\gamma_{i0} > 0$, $\hat{S}_i' = \text{diag}\{\hat{s}'_{i1}, \dots, \hat{s}'_{i,l_i-1}, 0\}$ with $\hat{s}'_{ik} = s'(\hat{v}_{ik}^T \bar{z}_i) = d[s(z_\alpha)]/dz_\alpha|_{z_\alpha = \hat{v}_{ik}^T \bar{z}_i}$, $k = 1, \dots, l_i - 1$, the NN node number $l_i > 1$, the residual term d_{ui} is bounded by

$$|d_{ui}| \leq \|V_i^*\|_F \|\bar{z}_i \hat{W}_i^T \hat{S}_i'\|_F + \|W_i^*\| \|\hat{S}_i' \hat{V}_i^T \bar{z}_i\| + \|W_i^*\|_1 \tag{76}$$

and the approximation error $\varepsilon_i(z_i)$ in (75) satisfies $|\varepsilon_i(z_i)| \leq \varepsilon_i^*$, $\forall z_i \in \Omega_{z_i}$ with constant $\varepsilon_i^* > 0$.

Consider the following control law:

$$v_i(t) = N(\zeta_i) \left[k_{i0} s_i + \hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i) + \hat{\delta}_i \xi_i(z_i) \tanh(s_i \xi_i(z_i)/\rho_i) \right] \tag{77}$$

$$\dot{\zeta}_i = k_{i0} s_i^2 + \hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i) s_i + \hat{\delta}_i \xi_i(z_i) s_i \tanh(s_i \xi_i(z_i)/\rho_i) \tag{78}$$

where $N(\zeta_i) = e^{\zeta_i^2} \cos((\pi/2)\zeta_i)$, k_{i0} , and ρ_i are positive constants, $\hat{\delta}_i$ is the estimate of δ_i^* with $\delta_i^* = \max\{\|V_i^*\|_F, \|W_i^*\|, \|W_i^*\|_1 + \varepsilon_i^* + p_i^*\}$ at time t , and

$$\xi_i(z_i) = \|\bar{z}_i \hat{W}_i^T \hat{S}_i'\|_F + \|\hat{S}_i' \hat{V}_i^T \bar{z}_i\| + 1. \tag{79}$$

The adaptive laws are employed as follows:

$$\dot{\hat{W}}_i = \Gamma_{wi} \left[(\hat{S}_i - \hat{S}_i' \hat{V}_i^T \bar{z}_i) s_i - \sigma_{wi} \hat{W}_i \right] \tag{80}$$

$$\dot{\hat{V}}_i = \Gamma_{vi} \left[\bar{z}_i \hat{W}_i^T \hat{S}_i' s_i - \sigma_{vi} \hat{V}_i \right] \tag{81}$$

$$\dot{\hat{\delta}}_i = \eta_i \left[s_i \xi_i(z_i) \tanh(s_i \xi_i(z_i)/\rho_i) - \sigma_i \hat{\delta}_i \right] \tag{82}$$

where $\Gamma_{wi} > 0, \Gamma_{vi} > 0, \sigma_{wi}, \sigma_{vi}, \eta_i$, and σ_i are strictly positive constants.

Theorem 2: Consider the closed-loop system consisting of the plant (1), control law (77), and adaptation laws (80)–(82). If Assumptions 1–5 hold, then for bounded initial conditions, the overall closed-loop neural control system is semiglobally stable in the sense that all signals in the closed-loop system are bounded, the parameter estimates

$$\begin{aligned}
& (\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \in \Omega_{wi} \\
& = \left\{ (\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \mid \|\tilde{W}_i\|^2 \leq \frac{2\mu_i}{\lambda_{\min}(\Gamma_{wi}^{-1})}, \right. \\
& \quad \left. \|\tilde{V}_i\|_F^2 \leq \frac{2\mu_i}{\lambda_{\min}(\Gamma_{vi}^{-1})}, |\tilde{\delta}_i|^2 \leq 2\eta_i \mu_i \right\} \tag{83}
\end{aligned}$$

and $\forall x_i(0) \in \Omega_{i0}$, the state vector

$$x_i \in \Omega_{ic} = \left\{ x_i \mid \|x_i - x_{id}\| \leq c_{i0} (1 + \|\Lambda_i\|) \|\omega_i(0)\| + \left[1 + \frac{(1 + \|\Lambda_i\|) c_{i0}}{\lambda_i} \right] \times \sqrt{2b_{i1}\mu_i}, \bar{x}_{id} \in \Omega_{id} \right\} \subset \Omega_i \quad (84)$$

where $\omega_i = [e_{i1}, \dots, e_{i,n_i-1}]^T \in R^{n_i-1}$, $\dot{\omega}_i = A_{s_i}\omega_i + b_{s_i}s_i$ is one state-space representation for mapping $s_i = [\Lambda_i^T 1]e_i$ with $\Lambda_i = [\lambda_{i1}, \dots, \lambda_{i,n_i-1}]^T$, $b_{s_i} = [0, \dots, 0, 1]^T \in R^{n_i-1}$, A_{s_i} being a stable matrix, and c_{i0} being a positive constant satisfying $\|e^{A_{s_i}t}\| \leq c_{i0}e^{-\lambda_i t}$, $i = 1, \dots, m$, and

$$\begin{aligned} \Omega_{i0} &= \{x_i(0) \mid \{x_i(t) \mid \|x_i(t) - x_{id}(t)\| < R_i(0, x_i(0), 0, 0, 0), \forall t \geq 0\} \\ &\subset \Omega_i, x_i(0) \in \Omega_i, \bar{x}_{id} \in \Omega_{id}\} \end{aligned} \quad (85)$$

with $c_i = \mu_{i0}/\lambda_{i0}$, and

$$R_i \left(c_i, x_i(0), \tilde{W}_i(0), \tilde{V}_i(0), \tilde{\delta}_i(0) \right) = c_{i0} (1 + \|\Lambda_i\|) \|\omega_i(0)\| + \left[1 + \frac{(1 + \|\Lambda_i\|) c_{i0}}{\lambda_i} \right] \sqrt{2b_{i1}\mu_i}. \quad (86)$$

Proof: The proof includes two steps. We first suppose that $x_i \in \Omega_i$ holds for all time, and find the upper bounds of system states. Later, for the appropriate initial condition $x_i(0)$ and the adaptive controller parameters, we prove that state x_i indeed remains in the compact set Ω_i for all $t \geq 0$.

Suppose that $x_i \in \Omega_i, \forall t \geq 0$, then NN approximation (75) is valid. Consider the Lyapunov function candidate

$$V_i(t) = V_{s_i}(t) + \frac{1}{2} \tilde{W}_i^T \Gamma_{w_i}^{-1} \tilde{W}_i + \frac{1}{2} \text{tr} \left\{ \tilde{V}_i^T \Gamma_{v_i}^{-1} \tilde{V}_i \right\} + \frac{1}{2\eta_i} \tilde{\delta}_i^2. \quad (87)$$

Differentiating $V_i(t)$ with respect to time t leads to

$$\dot{V}_i(t) = \dot{V}_{s_i}(t) + \tilde{W}_i^T \Gamma_{w_i}^{-1} \dot{\tilde{W}}_i + \text{tr} \left\{ \tilde{V}_i^T \Gamma_{v_i}^{-1} \dot{\tilde{V}}_i \right\} + \frac{1}{\eta_i} \tilde{\delta}_i \dot{\tilde{\delta}}_i. \quad (88)$$

Substituting inequality (69) into (88), and using (75) and (76) and control law (77) and (78), it follows that

$$\begin{aligned} \dot{V}_i(t) &\leq s_i g_i(t) v_i(t) + s_i h_i(z_i) + |s_i| p_i^* + \tilde{W}_i^T \Gamma_{w_i}^{-1} \dot{\tilde{W}}_i \\ &\quad + \text{tr} \left\{ \tilde{V}_i^T \Gamma_{v_i}^{-1} \dot{\tilde{V}}_i \right\} + \frac{1}{\eta_i} \tilde{\delta}_i \dot{\tilde{\delta}}_i \\ &= g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i - k_{i0} s_i^2 - \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) s_i \\ &\quad - \hat{\delta}_i \xi_i(z_i) s_i \tanh(s_i \xi_i(z_i) / \rho_i) \\ &\quad + \left[\hat{W}_i^T S \left(\hat{V}_i^T \bar{z}_i \right) - \tilde{W}_i^T \left(\hat{S}_i - \hat{S}_i^T \hat{V}_i^T \bar{z}_i \right) \right. \\ &\quad \left. - \hat{W}_i^T \hat{S}_i^T \hat{V}_i^T \bar{z}_i - d_{ui} + \varepsilon_i(z_i) \right] s_i \\ &\quad + |s_i| p_i^* + \tilde{W}_i^T \Gamma_{w_i}^{-1} \dot{\tilde{W}}_i + \text{tr} \left\{ \tilde{V}_i^T \Gamma_{v_i}^{-1} \dot{\tilde{V}}_i \right\} \\ &\quad + \frac{1}{\eta_i} \tilde{\delta}_i \dot{\tilde{\delta}}_i. \end{aligned} \quad (89)$$

Using adaptive tuning laws (80)–(82), and the inequality $0 \leq |r| - r \tanh(r/\varepsilon_0) \leq 0.2785\varepsilon_0$, for $\varepsilon_0 > 0$ and $r \in R$, and the fact that $\tilde{W}_i^T \hat{S}_i^T \hat{V}_i^T \bar{z}_i = \text{tr} \{ \tilde{V}_i^T \bar{z}_i \hat{W}_i^T \hat{S}_i \}$, we obtain

$$\begin{aligned} \dot{V}_i(t) &\leq g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i - k_{i0} s_i^2 + \delta_i^* \xi_i(z_i) |s_i| \\ &\quad - \delta_i^* \xi_i(z_i) s_i \tanh(s_i \xi_i(z_i) / \rho_i) - \sigma_{w_i} \tilde{W}_i^T \dot{\tilde{W}}_i \\ &\quad - \sigma_{v_i} \text{tr} \left\{ \tilde{V}_i^T \dot{\tilde{V}}_i \right\} - \sigma_i \tilde{\delta}_i \dot{\tilde{\delta}}_i \\ &\leq g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i - k_{i0} s_i^2 + 0.2785 \delta_i^* \rho_i \\ &\quad - \sigma_{w_i} \tilde{W}_i^T \dot{\tilde{W}}_i - \sigma_{v_i} \text{tr} \left\{ \tilde{V}_i^T \dot{\tilde{V}}_i \right\} - \sigma_i \tilde{\delta}_i \dot{\tilde{\delta}}_i. \end{aligned} \quad (90)$$

By completion of squares, the following inequalities hold:

$$-\sigma_{w_i} \tilde{W}_i^T \dot{\tilde{W}}_i \leq -\frac{\sigma_{w_i} \|\tilde{W}_i\|^2}{2} + \frac{\sigma_{w_i} \|W_i^*\|^2}{2} \quad (91)$$

$$-\sigma_{v_i} \text{tr} \left\{ \tilde{V}_i^T \dot{\tilde{V}}_i \right\} \leq -\frac{\sigma_{v_i} \|\tilde{V}_i\|_F^2}{2} + \frac{\sigma_{v_i} \|V_i^*\|_F^2}{2} \quad (92)$$

$$-\sigma_i \tilde{\delta}_i \dot{\tilde{\delta}}_i \leq -\frac{\sigma_i \tilde{\delta}_i^2}{2} + \frac{\sigma_i \delta_i^{*2}}{2}. \quad (93)$$

We obtain

$$\begin{aligned} \dot{V}_i(t) &\leq -k_{i0} s_i^2 + g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i + 0.2785 \delta_i^* \rho_i \\ &\quad - \frac{\sigma_{w_i} \|\tilde{W}_i\|^2}{2} - \frac{\sigma_{v_i} \|\tilde{V}_i\|_F^2}{2} - \frac{\sigma_i \tilde{\delta}_i^2}{2} + \frac{\sigma_{w_i} \|W_i^*\|^2}{2} \\ &\quad + \frac{\sigma_{v_i} \|V_i^*\|_F^2}{2} + \frac{\sigma_i \delta_i^{*2}}{2}. \end{aligned} \quad (94)$$

Define the following constants:

$$\lambda_{i0} = \min \{ 2k_{i0} b_{i0}, \sigma_{w_i} / \lambda_{\max}(\Gamma_{w_i}^{-1}), \sigma_{v_i} / \lambda_{\max}(\Gamma_{v_i}^{-1}), \sigma_i \eta_i \} \quad (95)$$

$$\mu_{i0} = 0.2785 \delta_i^* \rho_i + \frac{\sigma_{w_i} \|W_i^*\|^2}{2} + \frac{\sigma_{v_i} \|V_i^*\|_F^2}{2} + \frac{\sigma_i \delta_i^{*2}}{2}. \quad (96)$$

Thus, we have

$$\dot{V}_i(t) \leq -\lambda_{i0} V_i(t) + \mu_{i0} + g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i. \quad (97)$$

Multiplying inequality (97) by $e^{\lambda_{i0} t}$ yields

$$\frac{d}{dt} (V_i(t) e^{\lambda_{i0} t}) \leq e^{\lambda_{i0} t} \mu_{i0} + e^{\lambda_{i0} t} (g_i(t) N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i). \quad (98)$$

Integrating (98) over $[0, t]$, we have

$$\begin{aligned} 0 \leq V_i(t) &\leq \frac{\mu_{i0}}{\lambda_{i0}} + \left[V_i(0) - \frac{\mu_{i0}}{\lambda_{i0}} \right] e^{-\lambda_{i0} t} \\ &\quad + e^{-\lambda_{i0} t} \int_0^t (g_i(\tau) N(\zeta_i) + 1) \dot{\zeta}_i e^{\lambda_{i0} \tau} d\tau \\ &\leq \frac{\mu_{i0}}{\lambda_{i0}} + V_i(0) + e^{-\lambda_{i0} t} \int_0^t \\ &\quad \times (g_i(\tau) N(\zeta_i) + 1) \dot{\zeta}_i e^{\lambda_{i0} \tau} d\tau. \end{aligned} \quad (99)$$

From (10)–(15) and Assumption 3, we know that $g_i(t) \in [\min\{k_{i0}, k_{ir0}\}, k_{i11} + k_{ir1}] \subset (0, +\infty)$ or $g_i(t) \in [-k_{i11} + k_{ir1}, -\min\{k_{i0}, k_{ir0}\}] \subset (-\infty, 0)$. According to Lemma 2, we have that $V_i(t)$, $\zeta_i(t)$, and $\int_0^t g_i(\tau) N(\zeta_i) \dot{\zeta}_i d\tau$

are bounded in $[0, t_f)$. According to Lemma 3, we see that the above conclusion holds for $t_f = +\infty$. Therefore, $\hat{\delta}_i$, $\|\hat{W}_i\|$, and $\|\tilde{V}_i\|_F \in L_\infty$. Let C_{ζ_i} be the upper bound of $e^{-\lambda_{i0}t} \int_0^t (g_i(\tau)N(\zeta_i) + 1)\zeta_i e^{\lambda_{i0}\tau} d\tau$ in $[0, \infty)$, and

$$\mu_i = \frac{\mu_{i0}}{\lambda_{i0}} + V_i(0) + C_{\zeta_i} \quad (100)$$

then $s_i^2 \leq 2b_{i1}V_i(t) \leq 2b_{i1}\mu_i$. Similarly, $\|\tilde{W}_i\|^2 \leq 2\mu_i/\lambda_{\min}(\Gamma_{wi}^{-1})$, and $\|\tilde{V}_i\|_F^2 \leq 2\mu_i/\lambda_{\min}(\Gamma_{vi}^{-1})$, and $|\hat{\delta}_i|^2 \leq 2\eta_i\mu_i$.

Furthermore, similar to the discussion in Theorem 1, we can conclude that all the closed-loop signals are semiglobally uniformly ultimately bounded.

Remark 4: Since in control law $v_i(t)$ determined by (77), $\hat{\delta}_i$ is an adaptive bounding for unknown parameter δ^* including probably large values of ε_i^* , $\|V_i^*\|_F$, and $\|W_i^*\|$, then large magnitude switching could occur in the control signal. In order to deal with probably large values of ε_i^* , $\|V_i^*\|_F$, and $\|W_i^*\|$, a modified control scheme is presented as follows.

Consider the following control law:

$$v_i(t) = N(\zeta_i) \left[k_i(t)s_i + \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) \right] \quad (101)$$

$$\dot{\zeta}_i = k_i(t)s_i^2 + \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) s_i \quad (102)$$

where $N(\zeta_i) = e^{\zeta_i^2} \cos((\pi/2)\zeta_i)$, $k_i(t) = k_{i0} + (\xi_i^2(z_i)/2\sigma_i)$, k_{i0} , and σ_i are positive constants.

The adaptive laws are employed as follows:

$$\dot{\hat{W}}_i = \Gamma_{wi} \left[\left(\hat{S}_i - \hat{S}_i' \hat{V}_i^T \bar{z}_i \right) s_i - \sigma_{wi} \hat{W}_i \right] \quad (103)$$

$$\dot{\hat{V}}_i = \Gamma_{vi} \left[\bar{z}_i \hat{W}_i^T \hat{S}_i' s_i - \sigma_{vi} \hat{V}_i \right] \quad (104)$$

where $\Gamma_{wi} > 0$, $\Gamma_{vi} > 0$, σ_{wi} , and σ_{vi} are strictly positive constants.

Consider the Lyapunov function candidate

$$V_i(t) = V_{s_i} + \frac{1}{2} \tilde{W}_i^T \Gamma_{wi}^{-1} \tilde{W}_i + \frac{1}{2} \text{tr} \left\{ \tilde{V}_i^T \Gamma_{vi}^{-1} \tilde{V}_i \right\}.$$

Noting that $\delta_i^* \xi_i(z_i) |s_i| \leq (\sigma_i \delta_i^{*2}/2) + (\xi_i^2(z_i) s_i^2 / 2\sigma_i)$, it yields the following inequality:

$$\begin{aligned} \dot{V}_i(t) &\leq g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - k_i(t)s_i^2 + \delta_i^* \xi_i(z_i) |s_i| \\ &\quad - \sigma_{wi} \tilde{W}_i^T \hat{W}_i - \sigma_{vi} \text{tr} \left\{ \tilde{V}_i^T \hat{V}_i \right\} \\ &\leq g_i(t)N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - k_{i0}s_i^2 - \sigma_{wi} \tilde{W}_i^T \hat{W}_i \\ &\quad - \sigma_{vi} \text{tr} \left\{ \tilde{V}_i^T \hat{V}_i \right\} + \frac{\sigma_i \delta_i^{*2}}{2}. \end{aligned} \quad (105)$$

Therefore, employing the argument in Theorem 2, the similar conclusion can be obtained as well.

From the above two design schemes, we can see that large control efforts are required in this paper using either large magnitude switching or high gains in order to deal with probably large values of ε_i^* , $\|V_i^*\|_F$, and $\|W_i^*\|$.

IV. ADAPTIVE NN CONTROL WITH DEAD ZONES OF EQUAL SLOPES

Note that in Section III, adaptive neural control is designed for the system (1) with general nonlinear dead-zone inputs. In this section, we consider a special case of the system (1) with the dead zones of equal slopes. We make the following assumption.

Assumption 6: Suppose that the dead zones are described as follows:

$$u_i = D_i(v_i) = \begin{cases} k_{ir}(v_i - b_{ir}), & \text{if } v_i \geq b_{ir} \\ 0, & \text{if } b_{il} < v_i < b_{ir} \\ k_{il}(v_i - b_{il}), & \text{if } v_i \leq b_{il}. \end{cases}$$

The dead-zone parameters b_{ir} , b_{il} , k_{ir} , and k_{il} are unknown bounded constants, but their signs are known, i.e., $b_{ir} > 0$, $b_{il} < 0$, $k_{ir} > 0$, $k_{il} > 0$, and $k_{ir} = k_{il} = k_i$, $i = 1, \dots, m$.

In order to make full use of Assumption 6, the i th dead-zone output u_i can be denoted as follows:

$$u_i = k_i v_i + d_i(v_i) \quad (106)$$

where

$$d_i(v_i) = \begin{cases} -k_i b_{ir}, & \text{if } v_i \geq b_{ir} \\ -k_i v_i, & \text{if } b_{il} < v_i < b_{ir} \\ -k_i b_{il}, & \text{if } v_i \leq b_{il} \end{cases} \quad (107)$$

and $|d_i(v_i)| \leq p_i^*$, p_i^* is an unknown positive constant with $p_i^* = k_i \max\{b_{ir}, -b_{il}\}$.

For the i th subsystem of the MIMO nonlinear system (1), we design for each subsystem a full state feedback controller as follows. The closed-loop stability can be proved as a whole, which is similar to the proof of Theorem 2.

Consider the following control law:

$$v_i = N(\zeta_i) \left[k_{i0} s_i + \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) + \hat{\delta}_i \xi_i(z_i) \text{sgn}(s_i) \right] \quad (108)$$

$$\dot{\zeta}_i = k_{i0} s_i^2 + \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) s_i + \hat{\delta}_i \xi_i(z_i) |s_i| \quad (109)$$

where $N(\zeta_i) = e^{\zeta_i^2} \cos((\pi/2)\zeta_i)$, k_{i0} is a positive constant, $\hat{\delta}_i$ is the estimate of δ_i^* with $\delta_i^* = \max\{\|V_i^*\|_F, \|W_i^*\|, \|W_i^*\|_1 + \varepsilon_i^* + p_i^*\}$ at time t , and the sign function is defined as

$$\text{sgn}(z) = \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{if } z = 0 \\ -1, & \text{if } z < 0 \end{cases} \quad (110)$$

$$\xi_i(z_i) = \left\| \bar{z}_i \hat{W}_i^T \hat{S}_i' \right\|_F + \left\| \hat{S}_i' \hat{V}_i^T \bar{z}_i \right\| + 1. \quad (111)$$

The adaptive laws are employed as follows:

$$\dot{\hat{W}}_i = \Gamma_{wi} \left(\hat{S}_i - \hat{S}_i' \hat{V}_i^T \bar{z}_i \right) s_i \quad (112)$$

$$\dot{\hat{V}}_i = \Gamma_{vi} \bar{z}_i \hat{W}_i^T \hat{S}_i' s_i \quad (113)$$

$$\dot{\hat{\delta}}_i = \eta_i |s_i| \xi_i(z_i) \quad (114)$$

where $\Gamma_{wi} > 0$, $\Gamma_{vi} > 0$, and η_i is a strictly positive constant.

From (69) and (106), we know that

$$\dot{V}_{s_i}(t) \leq \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i s_i v_i + s_i h_i(z_i) + |s_i| p_i^*. \quad (115)$$

Substituting (115) into (88), and using (75) and (76) and control law (108) and (109), it follows that

$$\begin{aligned}
 \dot{V}_i(t) &\leq s_i \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i v_i + s_i h_i(z_i) + |s_i| p_i^* + \tilde{W}_i^T \Gamma_{wi}^{-1} \dot{W}_i \\
 &\quad + \text{tr} \left\{ \tilde{V}_i^T \Gamma_{vi}^{-1} \dot{V}_i \right\} + \frac{1}{\eta_i} \tilde{\delta}_i \dot{\delta}_i \\
 &= \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i - k_{i0} s_i^2 - \hat{W}_i^T S_i \left(\hat{V}_i^T \bar{z}_i \right) s_i \\
 &\quad - \hat{\delta}_i \xi_i(z_i) |s_i| \\
 &\quad + \left[\hat{W}_i^T S \left(\hat{V}_i^T \bar{z}_i \right) - \tilde{W}_i^T \left(\hat{S}_i - \hat{S}_i^T \hat{V}_i^T \bar{z}_i \right) \right. \\
 &\quad \quad \left. - \hat{W}_i^T \hat{S}_i^T \tilde{V}_i^T \bar{z}_i - d_{ui} + \varepsilon_i(z_i) \right] s_i \\
 &\quad + |s_i| p_i^* + \tilde{W}_i^T \Gamma_{wi}^{-1} \dot{W}_i + \text{tr} \left\{ \tilde{V}_i^T \Gamma_{vi}^{-1} \dot{V}_i \right\} \\
 &\quad + \frac{1}{\eta_i} \tilde{\delta}_i \dot{\delta}_i. \tag{116}
 \end{aligned}$$

Using adaptive tuning laws (112)–(114), and the fact that $\hat{W}_i^T \hat{S}_i^T \tilde{V}_i^T \bar{z}_i = \text{tr} \{ \tilde{V}_i^T \bar{z}_i \hat{W}_i^T \hat{S}_i^T \}$, we obtain

$$\dot{V}_i(t) \leq \frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i N(\zeta_i) \dot{\zeta}_i + \dot{\zeta}_i - k_{i0} s_i^2, \quad z_i \in \Omega_{z_i}. \tag{117}$$

Now, we will show that s_i^2 is uniformly continuous. Consider the resulting closed-loop system. There are three modes for its right-hand side corresponding to three cases: $s_i > 0$, $s_i < 0$, and $s_i = 0$ (sliding mode). Changing the sign of s_i implies changes in the motion law of the system and it does not make a jump in the system state x_i , therefore the system state is continuous, though the right-hand side of the differential equation is changed. In each mode, the system state x_i is continuous, and hence s_i is continuous in view of (17).

Integrating (117) over $[0, t]$, we have

$$\begin{aligned}
 0 \leq V_i(t) &\leq - \int_0^t k_{i0} s_i^2(\tau) d\tau + V_i(0) \\
 &\quad + \int_0^t \left(\frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i N(\zeta_i) + 1 \right) \dot{\zeta}_i d\tau \\
 &\leq V_i(0) + \int_0^t \left(\frac{b_i(\bar{x}_i)}{|b_i(\bar{x}_i)|} k_i N(\zeta_i) + 1 \right) \dot{\zeta}_i d\tau. \tag{118}
 \end{aligned}$$

According to Lemma 1, we have that $V_i(t)$, $\zeta_i(t)$, $\int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) \dot{\zeta}_i) d\tau$ are bounded in $[0, t_f)$. Note that $\int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) + 1) \dot{\zeta}_i d\tau = \int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) \dot{\zeta}_i) d\tau + \int_0^t \dot{\zeta}_i d\tau = \int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) \dot{\zeta}_i) d\tau + \zeta_i(t) - \zeta_i(0)$, and we obtain that $\int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) + 1) \dot{\zeta}_i d\tau$ is bounded in $[0, t_f)$ for bounded initial value $\zeta_i(0)$. According to Lemma 3, we see that the above conclusion is true for $t_f = +\infty$. Therefore, we know that $V_i(t)$, ζ_i , $\tilde{\delta}_i$, $\|\tilde{W}_i\|$, and $\|\tilde{V}_i\|_F \in L_\infty$, $s_i \in L_2$. Furthermore, it is easy to obtain x , e_i , and $\dot{s}_i \in L_\infty$. This means that s_i^2 is uniformly continuous in the interval $[0, \infty)$. According to Barbalat's lemma, we conclude that $\lim_{t \rightarrow \infty} s_i = 0$. From the discussion in [31], we have $\lim_{t \rightarrow \infty} e_{i1} = 0$. Let C_{ζ_i} be the upper bound of $\int_0^t ((b_i(\bar{x}_i)/|b_i(\bar{x}_i)|) k_i N(\zeta_i) + 1) \dot{\zeta}_i d\tau$ in $[0, \infty)$,

$\mu_i = V_i(0) + C_{\zeta_i}$, then $s_i^2 \leq 2b_{i1} V_i(t) \leq 2b_{i1} \mu_i$. Similarly, we obtain $\|\tilde{W}_i\|^2 \leq 2\mu_i/\lambda_{\min}(\Gamma_{wi}^{-1})$, $\|\tilde{V}_i\|_F^2 \leq 2\mu_i/\lambda_{\min}(\Gamma_{vi}^{-1})$, and $|\tilde{\delta}_i|^2 \leq 2\eta_i \mu_i$.

The following theorem shows the stability and control performance of the closed-loop adaptive system.

Theorem 3: Consider the closed-loop system consisting of the plant (1) under Assumptions 1 and 4–6, control law (108), and adaptation laws (112)–(114). For bounded initial conditions, the overall neural control system is semiglobally stable in the sense that all of the signals in the closed-loop system are bounded, the parameter estimates

$$\begin{aligned}
 (\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \in \Omega_{wi} &= \left\{ (\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \mid \|\tilde{W}_i\|^2 \leq \frac{2\mu_i}{\lambda_{\min}(\Gamma_{wi}^{-1})}, \right. \\
 &\quad \left. \|\tilde{V}_i\|_F^2 \leq \frac{2\mu_i}{\lambda_{\min}(\Gamma_{vi}^{-1})}, |\tilde{\delta}_i|^2 \leq 2\eta_i \mu_i \right\} \tag{119}
 \end{aligned}$$

the state vector

$$\begin{aligned}
 x_i \in \Omega_{ic} &= \left\{ x_i \mid \|x_i - x_{id}\| \leq c_{i0} (1 + \|\Lambda_i\|) \|\omega_i(0)\| \right. \\
 &\quad \left. + \left[1 + \frac{(1 + \|\Lambda_i\|) c_{i0}}{\lambda_i} \right] \sqrt{2b_{i1} \mu_i}, \bar{x}_{id} \in \Omega_{id} \right\} \tag{120}
 \end{aligned}$$

and the tracking errors converge to zero asymptotically, i.e., $\lim_{t \rightarrow \infty} e_{i1} = 0$.

Remark 5: Although the stability proofs in Sections III-C and IV is similar except the use of the sign function and the tanh function, and σ -modification terms, the asymptotic stability cannot be obtained with different slopes of the dead zone. In fact, if we adopt the same design scheme for the systems with general nonlinear dead zones similar to the adaptive control (108)–(114) without σ -modification and with the $\text{sgn}(\cdot)$ function, which is used in the control design, then we can obtain the following inequality:

$$\begin{aligned}
 0 \leq V_i(t) &\leq - \int_0^t k_{i0} s_i^2(\tau) d\tau + V_i(0) + \int_0^t (g_i(\tau) N(\zeta_i) + 1) \dot{\zeta}_i d\tau \\
 &\leq V_i(0) + \int_0^t (g_i(\tau) N(\zeta_i) + 1) \dot{\zeta}_i d\tau.
 \end{aligned}$$

Since $g_i(t)$ is a function of time t in the above equation, we know that the stability result cannot be obtained from the above inequality and Lemma 1 as well as Lemma 2. Therefore, for the dead zones with equal slopes, we can obtain asymptotic tracking result using adaptive control given by (108)–(114) from Lemma 1 and Theorem 3. However, we can only obtain the semiglobally stable result using adaptive control given by (77)–(82) for the general dead zones by Lemma 2 and Theorem 2.

Remark 6: Under weak conditions, we can only achieve ultimate boundedness of system trajectory as detailed in Theorems 1 and 2. However, under more strict conditions, we can achieve the asymptotic stability as stated in Theorem 3. Though we have obtained the asymptotic stability of the tracking error in Theorem 3 by introducing discontinuous sign function $\text{sgn}(s_i)$, the discontinuous control law (108) will lead to the ‘‘chattering’’

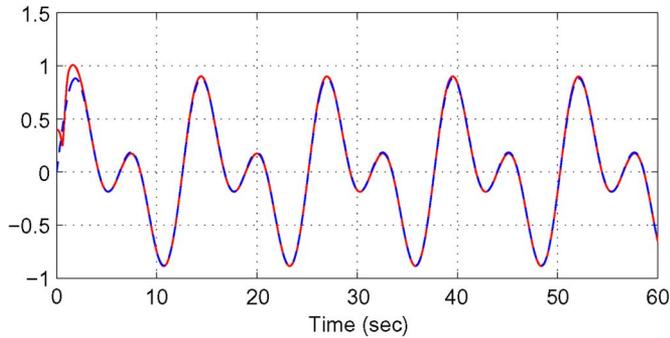


Fig. 3. Output y_1 (solid line) follows desired trajectory y_{1d} (dashed line) using control law (77) with nonsymmetric dead zones.

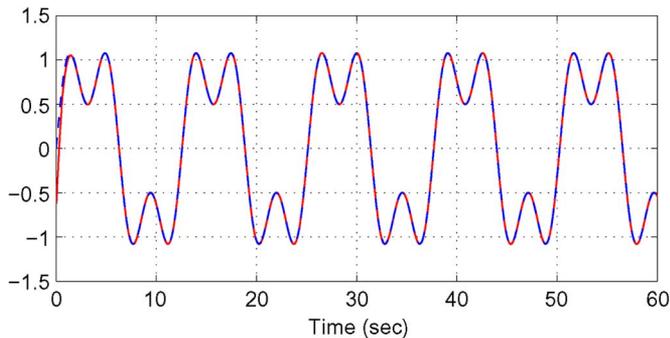


Fig. 4. Output y_2 (solid line) follows desired trajectory y_{2d} (dashed line) using control law (77) with nonsymmetric dead zones.

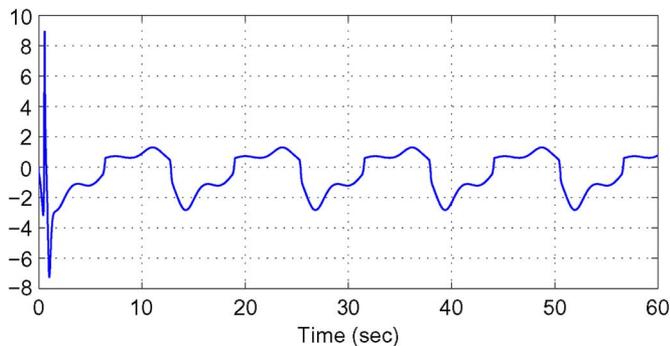


Fig. 5. Control signal v_1 using control law (77) with nonsymmetric dead zones.

phenomenon in control. This is also verified in the simulations: good tracking performance, but chattering control signals. Chattering may be undesirable in practice. This phenomenon can be eliminated using controller (77) or (101). But the asymptotic stability will disappear.

V. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed approach, the developed adaptive NN tracking controller is applied to the following nonlinear system:

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = x_{21} - 0.3 \sin(x_{12}) + b_1(x_{11})u_1 \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = x_{11}^2 u_1 + (x_{22}^2 + x_{11} + 0.5 \cos(x_{21})) u_2 + b_2(x_{22})u_2 \\ y_1 = x_{11}, y_2 = x_{21} \end{cases} \quad (121)$$

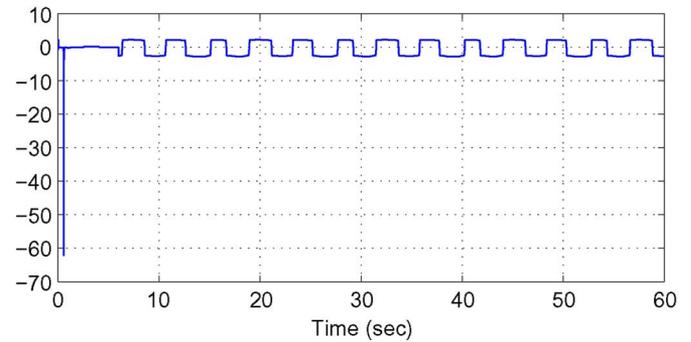


Fig. 6. Control signal v_2 using control law (77) with nonsymmetric dead zones.

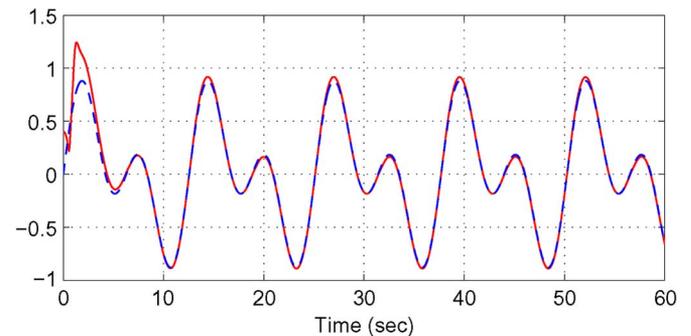


Fig. 7. Output y_1 (solid line) follows desired trajectory y_{1d} (dashed line) using control law (101) with nonsymmetric dead zones and positive gains.

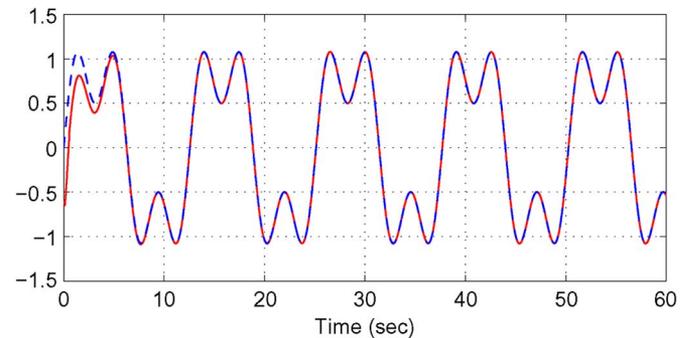


Fig. 8. Output y_2 (solid line) follows desired trajectory y_{2d} (dashed line) using control law (101) with nonsymmetric dead zones and positive gains.

where $b_1(x_{11}) = 2 - \sin^2(x_{11})$ and $b_2(x_{22}) = 3 + \sin(x_{22})$, u_i ($i = 1, 2$) are the outputs of the dead zones. The control objective is to make the system outputs y_i ($i = 1, 2$) follow the desired trajectory y_{id} ($i = 1, 2$).

Both NNs $\hat{W}_i^T S_i(\hat{V}_i^T \bar{z}_i)$, $i = 1, 2$, contain ten hidden nodes (i.e., $l_1 = l_2 = 10$) and the coefficients in activation function $s(\cdot)$ are taken as $\gamma_{10} = \gamma_{20} = 3.5$. The desired tracking trajectories are $y_{1d}(t) = 0.5[\sin(t) + \sin(0.5t)]$ and $y_{2d} = \sin(0.5t) + 0.5 \sin(1.5t)$. The parameters of the dead zones are $k_{1l} = 0.5$, $k_{1r} = 1.5$, $k_{2l} = 1.5$, $k_{2r} = 2.5$, $b_{1l} = -0.5$, $b_{1r} = 0.5$, $b_{2l} = -2.5$, and $b_{2r} = 2$. The control law is given by (77), and the adaptive laws are determined by (80)–(82). The design parameters of the above adaptive control are chosen as $\lambda_1 = 1.5$, $\lambda_2 = 2$, $k_{10} = 2$, $k_{20} = 5$, $\eta_1 = \eta_2 = 0.1$, $\rho_1 = \rho_2 = 0.2$, $\sigma_1 = \sigma_2 = \sigma_{w1} = \sigma_{w2} = \sigma_{v1} = \sigma_{v2} = 0.01$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}\{2.5\}$, and $\Gamma_{v1} = \Gamma_{v2} = \text{diag}\{10\}$. The

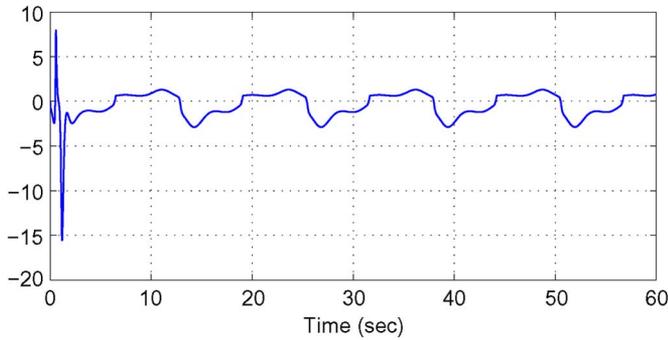


Fig. 9. Control signal v_1 using control law (101) with nonsymmetric dead zones and positive gains.

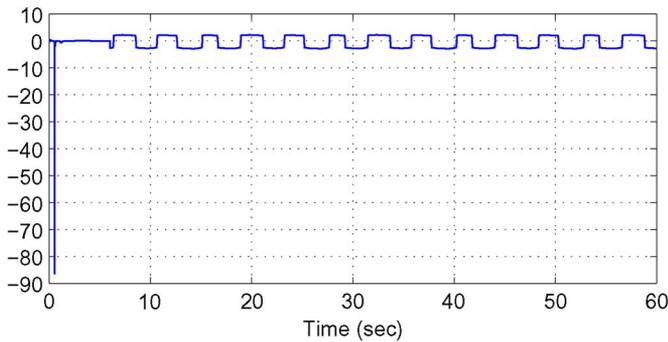


Fig. 10. Control signal v_2 using control law (101) with nonsymmetric dead zones and positive gains.

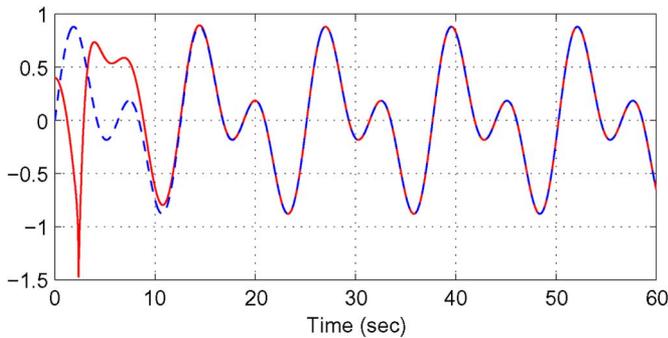


Fig. 11. Output y_1 (solid line) follows desired trajectory y_{1d} (dashed line) using control law (101) with nonsymmetric dead zones and negative gains.

initial conditions are $x_{11}(0) = 0.4$, $x_{12}(0) = 0$, $x_{21}(0) = -0.6$, $x_{22}(0) = 0$, $\hat{\delta}_i(0) = 0.5$, and $\zeta_i(0) = 0.5$, $i = 1, 2$. $\hat{W}_1(0)$ and $\hat{W}_2(0)$ are taken as zeros; $\hat{V}_1(0)$ and $\hat{V}_2(0)$ are randomly taken in the intervals $[-1.0, 1.0]$ and $[-0.5, 0.5]$, respectively, $\hat{\delta}_1(0)$ and $\hat{\delta}_2(0)$ are randomly taken in the interval $[0, 0.5]$. The simulation results are shown in Figs. 3–6.

If we employ control law (101), adaptive laws (103) and (104), $\lambda_1 = \lambda_2 = 0.5$, $k_{10} = k_{20} = 0.1$, $\sigma_1 = 0.5$, $\sigma_2 = 0.2$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}\{2.5\}$, and $\Gamma_{v1} = \Gamma_{v2} = \text{diag}\{2.5\}$, and all other conditions being the same as in Case 1, then the simulation results are shown in Figs. 7–10.

If we choose $b_1(x_{11}) = -(2 - \sin^2(x_{11}))$, $b_2(x_{22}) = -(3 + \sin(x_{22}))$, control law (101), adaptive laws (103) and (104), $\sigma_1 = 5$, $\sigma_2 = 2$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}\{0.1\}$, and all other con-

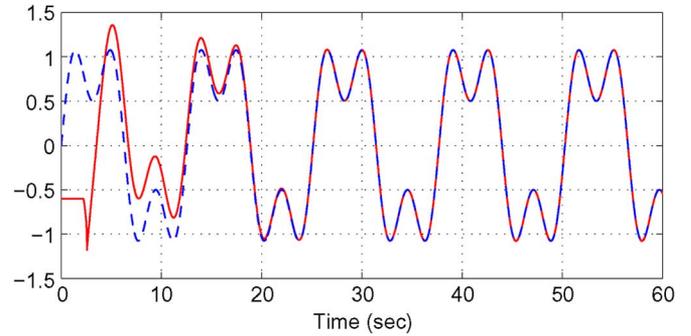


Fig. 12. Output y_2 (solid line) follows desired trajectory y_{2d} (dashed line) using control law (101) with nonsymmetric dead zones and negative gains.

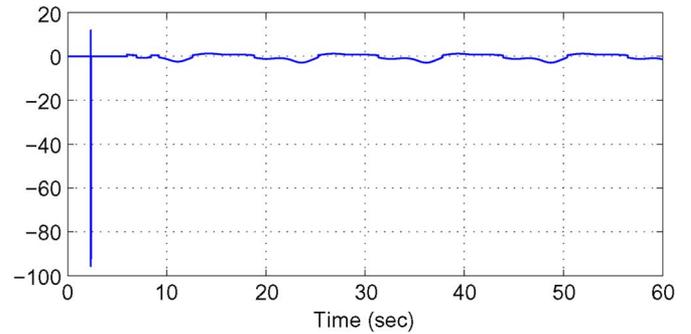


Fig. 13. Control signal v_1 using control law (101) with nonsymmetric dead zones and negative gains.

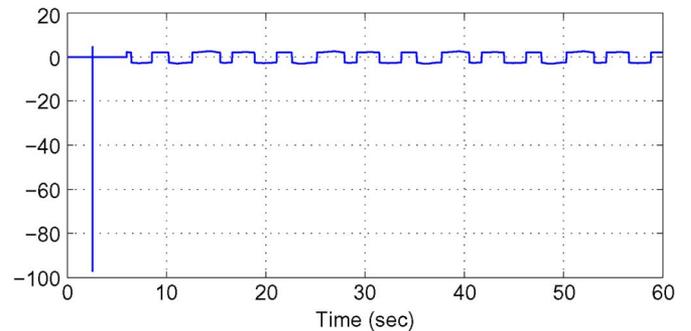


Fig. 14. Control signal v_2 using control law (101) with nonsymmetric dead zones and negative gains.

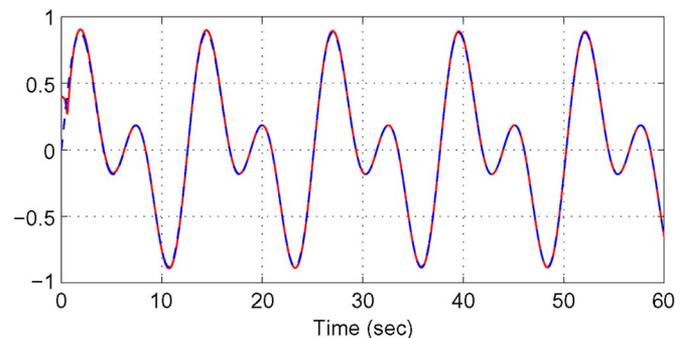


Fig. 15. Output y_1 (solid line) follows desired trajectory y_{1d} (dashed line) using control law (108) with equal slopes.

ditions being the same as in Case 2, then the simulation results are shown in Figs. 11–14.

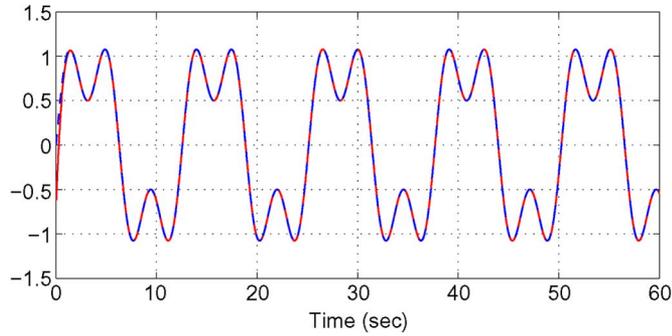


Fig. 16. Output y_2 (solid line) follows desired trajectory y_{2d} (dashed line) using control law (108) with equal slopes.

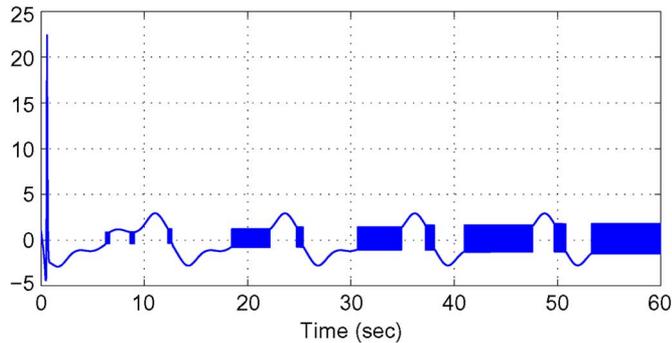


Fig. 17. Control signal v_1 using control law (108) with equal slopes.

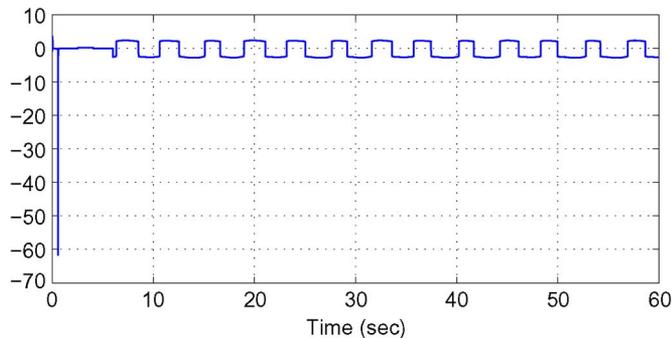


Fig. 18. Control signal v_2 using control law (108) with equal slopes.

If we select $k_{1l} = k_{1r} = 0.5$, $k_{2l} = k_{2r} = 1.5$, control law (108), adaptive laws (112)–(114), $\lambda_1 = \lambda_2 = 2.5$, $k_{10} = 4$, $k_{20} = 5$, $\eta_1 = \eta_2 = 0.1$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}\{0.1\}$, $\Gamma_{v1} = \Gamma_{v2} = \text{diag}\{2.5\}$, and all other conditions being the same as in Case 1, the simulation results are shown in Figs. 15–18.

Figs. 3–18 clearly show that the system outputs and control signals are bounded for system (121) with the dead zones. From Figs. 3, 4, 7, 8, 11, 12, 15, and 16, it can be seen that fairly good tracking performance is obtained. Fig. 17 shows that the control signal v_1 happens to chattering phenomenon due to using discontinuous sign function $\text{sgn}(s_1)$.

VI. CONCLUSION

Three adaptive NN tracking control schemes have been presented for a class of uncertain MIMO nonlinear systems with

unknown dead zones and control directions. Controller singularity problems have been solved by employing integral Lyapunov function. Based on the intuitive concept and piecewise description of a dead zone and the principle of sliding mode control, the developed controller can guarantee that all signals involved are semiglobally uniformly ultimately bounded. The proposed approach does not require the bounds of the dead-zone parameters to be known. Moreover, the functions outside the dead band may be nonlinear functions.

ACKNOWLEDGMENT

The authors would like to thank T.T. Han and S.-L. Dai for their inputs that improved this paper.

REFERENCES

- [1] R. M. Sanner and J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Trans. Neural Netw.*, vol. 3, no. 6, pp. 837–863, Nov. 1992.
- [2] C. Y. Su and Y. Stepanenko, "Adaptive control of a class of nonlinear systems with fuzzy logic," *IEEE Trans. Fuzzy Syst.*, vol. 2, no. 3, pp. 285–294, Jun. 1994.
- [3] A. Yesildirek and F. L. Lewis, "Feedback linearization using neural networks," *Automatica*, vol. 31, no. 11, pp. 1659–1664, 1995.
- [4] T. Zhang, S. S. Ge, and C. C. Hang, "Stable adaptive control for a class of nonlinear systems using a modified Lyapunov function," *IEEE Trans. Autom. Control*, vol. 45, no. 1, pp. 129–132, Jan. 2000.
- [5] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*. Boston, MA: Kluwer, 2001.
- [6] S. S. Ge, T. Zhang, and C. C. Hang, "Stable adaptive control for nonlinear multivariable systems with a triangular control structure," *IEEE Trans. Autom. Control*, vol. 45, no. 6, pp. 1221–1224, Jun. 2000.
- [7] T. P. Zhang, "Direct adaptive neural network control based on integral Lyapunov function," *Acta Automatica Sinica*, vol. 29, no. 6, pp. 996–1001, 2003.
- [8] T. P. Zhang, "Adaptive fuzzy sliding mode control based on a modified Lyapunov function," *Acta Automatica Sinica*, vol. 28, no. 1, pp. 137–142, 2002.
- [9] T. P. Zhang, "Stable adaptive fuzzy sliding mode control of interconnected systems," *Fuzzy Sets Syst.*, vol. 122, no. 1, pp. 5–19, 2001.
- [10] T. P. Zhang, "Stable direct adaptive fuzzy control for a class of MIMO nonlinear systems," *Int. J. Syst. Sci.*, vol. 34, no. 6, pp. 375–388, 2003.
- [11] R. D. Nussbaum, "Some remarks on the conjecture in parameter adaptive control," *Syst. Control Lett.*, vol. 3, no. 3, pp. 243–246, 1983.
- [12] X. D. Ye and J. P. Jiang, "Adaptive nonlinear design without a priori knowledge of control directions," *IEEE Trans. Autom. Control*, vol. 43, no. 11, pp. 1617–1621, Nov. 1998.
- [13] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural control of nonlinear time-delay system with unknown virtual control coefficients," *IEEE Trans. Syst. Man Cybern. B, Cybern.*, vol. 34, no. 1, pp. 499–516, Feb. 2004.
- [14] G. Tao and P. V. Kokotovic, "Adaptive sliding control of plants with unknown dead-zone," *IEEE Trans. Autom. Control*, vol. 39, no. 1, pp. 59–68, Jan. 1994.
- [15] G. Tao and P. V. Kokotovic, "Discrete-time adaptive control of systems with unknown dead-zone," *Int. J. Control*, vol. 61, no. 1, pp. 1–17, 1995.
- [16] H. Y. Cho and E. W. Bai, "Convergence results for an adaptive dead zone inverse," *Int. J. Adapt. Control Signal Process.*, vol. 12, no. 5, pp. 451–466, 1998.
- [17] A. Taware and G. Tao, "An adaptive dead-zone inverse controller for systems with sandwiched dead-zones," *Int. J. Control*, vol. 76, no. 8, pp. 755–769, 2003.
- [18] R. R. Selmic and F. L. Lewis, "Deadzone compensation in motion control systems using neural networks," *IEEE Trans. Autom. Control*, vol. 45, no. 4, pp. 602–613, Apr. 2000.
- [19] X. S. Wang, H. Hong, and C. Y. Su, "Model reference adaptive control of continuous time systems with an unknown dead-zone," *Inst. Electr. Eng. Proc.—Control Theory Appl.*, vol. 150, no. 3, pp. 261–266, 2003.
- [20] X. S. Wang, H. Hong, and C. Y. Su, "Robust adaptive control a class of nonlinear systems with an unknown dead-zone," *Automatica*, vol. 40, no. 3, pp. 407–413, 2004.

- [21] K. K. Shyu, W. J. Liu, and K. C. Hsu, "Design of large-scale time-delayed systems with dead-zone input via variable structure control," *Automatica*, vol. 41, no. 7, pp. 1239–1246, 2005.
- [22] E. P. Ryan, "A nonlinear universal servomechanism," *IEEE Trans. Autom. Control*, vol. 39, no. 4, pp. 753–761, Apr. 1994.
- [23] J. Zhou, C. Y. Wen, and Y. Zhang, "Adaptive output control of nonlinear systems with uncertain dead-zone nonlinearity," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 504–511, Mar. 2006.
- [24] T. P. Zhang and S. S. Ge, "Robust adaptive neural control of SISO nonlinear systems with unknown nonlinear dead-zone and gain sign," in *Proc. IEEE Symp. Intell. Control*, Munich, Germany, Oct. 4–6, 2006, pp. 315–320.
- [25] J. Zhou, C. Y. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown dead-zone," in *Proc. IEEE Conf. Robot. Autom. Mechatron.*, Singapore, 2004, pp. 513–518.
- [26] E. P. Ryan, "A universal adaptive stabilizer for a class of nonlinear systems," *Syst. Control Lett.*, vol. 16, no. 3, pp. 209–218, 1991.
- [27] F. L. Lewis, A. Yesildirek, and K. Liu, "Multilayer neural network robot controller with guaranteed tracking performance," *IEEE Trans. Neural Netw.*, vol. 7, no. 2, pp. 388–398, Mar. 1996.
- [28] T. Zhang, S. S. Ge, and C. C. Hang, "Design and performance analysis of a direct adaptive controller for nonlinear systems," *Automatica*, vol. 35, no. 11, pp. 1809–1817, 1999.
- [29] T. P. Zhang and S. S. Ge, "Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs," *Automatica*, vol. 43, no. 6, pp. 1021–1033, 2007.
- [30] J. Zhou, M. J. Er, and K. C. Veluvolu, "Adaptive output control of nonlinear time-delayed systems with uncertain dead-zone input," in *Proc. Amer. Control Conf.*, Minneapolis, MN, Jun. 14–16, 2006, pp. 5312–5316.
- [31] J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.



Tianping Zhang received the B.S. degree in mathematics from Yangzhou Teachers College, Yangzhou, China, in 1986, the M.S. degree in operations research and control theory from East China Normal University, Shanghai, China, in 1992, and the Ph.D. degree in automatic control theory and applications from Southeast University, Nanjing, China, in 1996.

Currently, he is a Professor at the College of Information Engineering, Yangzhou University. From October 2005 to October 2006, he was a Visiting Scientist at the Department of Electrical and Computer Engineering, National University of Singapore, Singapore. His current research interests include fuzzy control, adaptive control, and nonlinear control.



Shuzhi Sam Ge (S'90–M'92–SM'00–F'06) received the B.Sc. degree from Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China, in 1986 and the Ph.D. degree and the Diploma of Imperial College (DIC) from Imperial College of Science, Technology and Medicine, London, U.K., in 1993.

He is the Director of Social Robotics Lab of Interactive Digital Media Institute, and Professor of Department of Electrical and Computer Engineering, the National University of Singapore, Singapore.

He (co)authored three books: *Adaptive Neural Network Control of Robotic Manipulators* (Singapore: World Scientific, 1998), *Stable Adaptive Neural Network Control* (Norwell, MA: Kluwer, 2001), and *Switched Linear Systems: Control and Design* (New York: Springer-Verlag, 2005). He edited a book *Autonomous Mobile Robots: Sensing, Control, Decision Making and Applications* (New York: Taylor & Francis, 2006), and wrote over 300 international journal and conference papers. His current research interests include social robotics, multimedia fusion, adaptive control, and intelligent systems.

Dr. Ge has served/been serving as an Associate Editor for a number of flagship journals including the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, the IEEE TRANSACTIONS ON NEURAL NETWORKS, and *Automatica*. He also serves as an editor of the Taylor & Francis Automation and Control Engineering Series, and as an elected member of Board of Governors, IEEE Control Systems Society.