

# Short Papers

## Robust Motion/Force Control of Uncertain Holonomic/Nonholonomic Mechanical Systems

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**Abstract**—In this short paper, robust control strategies are presented systematically for both holonomic mechanical systems and a large class of nonholonomic mechanical systems in the presence of uncertainties and disturbances. First, robust control strategies are presented for both kinds of systems using the bounds of system parameters, respectively. Then, adaptive robust control strategies are presented by tuning the parameter estimates online. Proportional plus integral feedback control is used for force control for the benefit of real-time implementation. The proposed control strategies guarantee that the system motion converges to the desired manifold with prescribed performance while the constraint force remains bounded.

**Index Terms**—Force control, holonomic, motion control, nonholonomic, robust.

### I. INTRODUCTION

In recent years, much attention has been devoted to the problem of controlling mechanical systems with holonomic or nonholonomic constraints. Both trajectory tracking and force control are manageable with a constrained robot if the exact robot dynamic model is available for controller design [1]. In real applications, however, perfect cancellation of the robot dynamics is almost impossible. Thanks to the researches in [2]–[4] and among others, the motion control part can be reduced to a problem similar to free-motion control of a robot with less degrees of freedom. Elegant robust adaptive motion controllers were proposed in [5] and [6] using the linear-in-parameters' property of the system dynamics and the bound of the robot parameters. Force control, however, remains a difficult problem. The use of force feedback was considered in [4] and [7] to obtain a bounded force error with a zero tracking error. Adaptive control combined with persistence excitation condition was studied in [8] to achieve zero force error. A nonlinear coordinate transformation was proposed in [9] for exact force control. However, the corresponding regressor may become complicated and difficult to implement. Parallel regulation was presented in [10] to guarantee zero force error at the expense of a steady-state position error.

At the same time, considerable attention has been paid to the motion control of nonholonomic mechanical systems during last few years [3], [11]–[13]. Model based control was reported in [14] to guarantee the asymptotic convergence of the motion errors and the boundedness of the force error. To remove the drawbacks associated with the regressor matrix based approaches, such as time-consuming computation of the regressor matrix, nonregressor matrix based control laws have been proposed. Learning control was presented in [15], but it is applicable only when the task is repetitive. In [16], an adaptive fuzzy approach was presented. But the asymptotic convergence property depends on a strong assumption that the approximation error plus external disturbance belong to  $L_2 \cap L_\infty$ . Furthermore, the tracking accuracy depends on the choice of the number of the fuzzy sets, which may lead to large

computational burden in practice. On the other hand, although theoretically the force error in these works can be made small by using a large proportional force feedback gain, the gain for the proportional force feedback is severely limited in applications due to the acausality problem and signal to noise ratio in real time implementation [17]. It is suggested that the best force tracking performance is achieved by integral (I) force feedback or proportional plus integral (PI) force feedback [18].

In this paper: 1) robust control strategies are proposed for both holonomic mechanical systems and a large class of nonholonomic systems in the presence of uncertainties and external disturbances. The controller design is developed in a systematic and unified manner for the two kinds of mechanical systems; 2) the controller design is nonregressor based and is carried out without imposing any restriction on the system dynamics; 3) adaptive technique is applied to relax the requirement of known bounds for the robust control design; and 4) proportional plus integral force feedback control is used to remove the severe limitation on proportional feedback gain arose by using proportional control only. The proposed control strategies guarantee that the system motion converges to the desired manifold with prescribed performance with bounded constraint force.

### II. SYSTEM DESCRIPTIONS

According to the Euler-Lagrangian formulation, the dynamics of an  $n$ -dimensional constrained mechanical system can be described as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \quad (1)$$

where  $q = [q_1, \dots, q_n]^T \in R^n$  denotes the vector of generalized coordinates;  $D(q) \in R^{n \times n}$  is the symmetric bounded positive definite inertia matrix;  $C(\dot{q}, q)\dot{q} \in R^n$  denotes the Centripetal and Coriolis torques;  $G(q) \in R^n$  is the gravitational torque vector;  $d$  denotes the external disturbances;  $\tau \in R^m$  is the vector of control input;  $B \in R^{n \times m}$  is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of the system; and  $f \in R^n$  denotes the vector of constraint forces.

*Property 1:* Matrices  $D(q)$ ,  $G(q)$  are uniformly bounded and uniformly continuous if  $q$  is uniformly bounded and continuous, respectively. Matrix  $C(q, \dot{q})$  is uniformly bounded and uniformly continuous if  $\dot{q}$  is uniformly bounded and continuous, respectively.

*Property 2:* There exist some finite positive constants  $c_i > 0$  ( $1 \leq i \leq 4$ ) and finite nonnegative constants  $c_i \geq 0$  ( $i = 5$ ) such that  $\forall q \in R^n, \forall \dot{q} \in R^n, \|D(q)\| \leq c_1, \|C(q, \dot{q})\| \leq c_2 + c_3\|\dot{q}\|, \|G(q)\| \leq c_4$ , and  $\sup_{t \geq 0} \|d(t)\| \leq c_5$ .

#### A. Holonomic Constrained Mechanical Systems

When the system is subjected to holonomic constraints, the  $l$  independent constraints can be expressed as

$$h(q) = 0 \in R^l. \quad (2)$$

For such systems, the dimension of the control input equals the dimension of the system coordinates, i.e.,  $m = n$ , and the matrix  $B$  is nonsingular. The constraint force,  $f \in R^n$ , is measured by a force sensor mounted on the end-effector and can be converted to the joint space as  $f = J^T(q)\lambda$ ,  $J(q) = (\partial h / \partial q) \in R^{l \times n}$  and  $\lambda \in R^l$  is a generalized Lagrangian multiplier. Hence, the holonomic constraint on the robot's

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end effector can be viewed as restricting only the dynamics on the constraint manifold  $\Omega_h$  defined by  $\Omega_h = \{(q, \dot{q}) \mid h(q) = 0, J(q)\dot{q} = 0\}$ .

The vector  $q \in R^n$  can always be properly rearranged and partitioned into  $q = [q^1 \ q^2]^T$ ,  $q^1 = [q_1^1 \ \dots \ q_{n-l}^1] \in R^{n-l}$  describes the constrained motion of the manipulator and  $q^2 = [q_1^2 \ \dots \ q_l^2] \in R^l$  denotes the remaining joint variables. There always exists a matrix  $L(q^1) \in R^{n \times (n-l)}$  such that  $\dot{q} = L(q^1)\dot{q}^1$  and  $L^T(q^1)J^T(q^1) = 0$ .

The dynamic model (1), when restricted to the constraint surface, can be expressed in the reduced form as

$$D(q^1)L(q^1)\ddot{q}^1 + C_1(q^1, \dot{q}^1)\dot{q}^1 + G(q^1) + d = B\tau + J^T(q^1)\lambda \quad (3)$$

where  $C_1 = D(q^1)\dot{L}(q^1) + C(q^1, \dot{q}^1)L(q^1)$ .

### B. Nonholonomic Constrained Mechanical Systems

When the system is subjected to nonholonomic constraints, the matrix  $B(q)$  is of full row rank with  $n - l \leq m \leq n$ , and the  $l$  nonintegrable and independent velocity constraints can be expressed as

$$J(q)\dot{q} = 0 \quad (4)$$

where  $J(q) = [J_1^T(q), \dots, J_l^T(q)]^T: R^n \rightarrow R^{l \times n}$  is the kinematic constraint matrix which is assumed to have full rank  $l$ . In this short paper, constraint (4) is assumed to be completely nonholonomic and exactly known. The effect of the constraints can be viewed as restricting the dynamics on the manifold  $\Omega_{nh}$  as  $\Omega_{nh} = \{(q, \dot{q}) \mid J(q)\dot{q} = 0\}$ .

The generalized constraint forces in mechanical system (1) can be given by  $f = J^T(q)\lambda$ , where  $\lambda \in R^l$  is known as force on the contact point between the rigid body and environmental surfaces.

Assume that the annihilator of the co-distribution spanned by the covector fields  $J_1(q), \dots, J_l(q)$  is an  $(n - l)$ -dimensional smooth nonsingular distribution  $\Delta$  on  $R$ . This distribution  $\Delta$  is spanned by a set of  $(n - l)$  smooth and linearly independent vector fields  $H_1(q), \dots, H_{n-l}(q)$ , i.e.,  $\Delta = \text{span}\{H_1(q), \dots, H_{n-l}(q)\}$ . Then  $H^T(q)J^T(q) = 0$ , where  $H(q) = [H_1(q), \dots, H_{n-l}(q)] \in R^{n \times (n-l)}$ . Note that  $H^T H$  is of full rank. Constraints (4) implies the existence of vector  $\dot{z} \in R^{n-l}$ , such that  $\dot{q} = H(q)\dot{z}$ .

The dynamic equation (1), which satisfies the nonholonomic constraint (4), can be rewritten in terms of the internal state variable  $\dot{z}$  as [14]

$$D(q)H(q)\ddot{z} + C_2(q, \dot{z})\dot{z} + G(q) + d = B(q)\tau + J^T(q)\lambda \quad (5)$$

where  $C_2(q, \dot{z}) = D(q)\dot{H}(q) + C(q, \dot{q})H(q)$ .

By exploiting the structure of the dynamic equations (3) and (5), some properties are listed as follows [8], [14], [16].

*Property 3:* The matrices  $D_L = L^T D L$  and  $D_H = H^T D H$  are symmetric and positive definite.

*Property 4:* The matrices  $\dot{D}_L - 2L^T C_1$  and  $\dot{D}_H - 2H^T C_2$  are skew-symmetric [19].

*Property 5:* For holonomic systems, matrices  $J(q^1)$ ,  $L(q^1)$  are uniformly bounded and uniformly continuous if  $q^1$  is uniformly bounded and continuous, respectively. For nonholonomic systems,  $D(q)$ ,  $G(q)$ ,  $J(q)$ , and  $H(q)$  are bounded and continuous if  $z$  is bounded and uniformly continuous.  $C(q, \dot{q})$  and  $\dot{H}(q)$  are bounded if  $\dot{z}$  is bounded.  $C(q, \dot{q})$  and  $\dot{H}(q)$  are uniformly continuous if  $\dot{z}$  is uniformly continuous.

## III. ROBUST CONTROL DESIGN

For clarity, define  $\lambda_{\min}(\ast)$  and  $\lambda_{\max}(\ast)$  as the smallest eigenvalue and the largest eigenvalue of  $(\ast)$ , respectively.

### A. Robust Control of Holonomic Mechanical Systems

The control objective is specified as: Given a desired joint trajectory  $q_d(t)$  and a desired constraint force  $f_d(t)$ , or, equivalently, a desired multiplier  $\lambda_d(t)$ , determine a control law such that for any  $(q(0), \dot{q}(0)) \in \Omega_h$ ,  $q^1$ ,  $\dot{q}$  and  $\lambda$  asymptotically converge to a manifold  $\Omega_{hd}$  specified as

$$\Omega_{hd} = \{(q, \dot{q}, \lambda) \mid q^1 = q_d^1, \dot{q} = L(q)\dot{q}_d^1, \lambda = \lambda_d\}. \quad (6)$$

*Assumption 1:* The desired reference trajectory  $q_d(t)$  is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired Lagrangian multiplier  $\lambda_d(t)$  is also bounded and uniformly continuous.

For holonomic systems, there is a unique function  $v: R^{n-m} \rightarrow R^m$  such that the constraint equation (2) can always be expressed explicitly as  $q^2 = v(q^1)$ . Thus it is only required to evolve a control law such that  $q^1(t) \rightarrow q_d^1(t)$  as  $t \rightarrow \infty$ .

Define  $e_{q^1} = q^1 - q_d^1$ ,  $e_\lambda = \lambda - \lambda_d$ ,  $\dot{q}_r^1 = \dot{q}_d^1 - \rho_1 e_{q^1}$ , and  $s = \dot{e}_{q^1} + \rho_1 e_{q^1}$  with  $\rho_1 > 0$ .

Consider the control law given by

$$B\tau = -K L s - \frac{L^+ s \Phi^2}{\|s\| \Phi + \delta} - J^T \lambda_c \quad (7)$$

where  $K$  is positive definite,  $L^+$  is the left inverse of  $L^T$  defined as  $L^+ = L(L^T L)^{-1}$ ,  $\lambda_c = \lambda_d - K_\lambda e_\lambda - K_I \int e_\lambda dt$ ,  $K_\lambda$  is a constant matrix of proportional control feedback gains,  $K_I$  is a constant matrix of integral control feedback gains, and  $\Phi = \|L(q^1)\| \{c_1 \|(d/dt)[L(q^1)\dot{q}_r^1]\| + (c_2 + c_3 \|\dot{q}\|) \|L(q^1)\dot{q}_d^1\| + c_4 + c_5\}$ , where  $\delta(t)$  is a time varying positive function converging to zero as  $t \rightarrow \infty$ , such that  $\int_0^t \delta(\omega) d\omega = a < \infty$ . There are many choices for  $\delta(t)$  that satisfies the condition. For example,  $\delta(t)$  may be chosen as  $(1+t)^{-l_1}$  ( $l_1 > 1$ ) or  $e^{-l_2 t}$  ( $l_2 > 0$ ).

Define  $\nu = \lambda_{\min}(L^T K L) / \lambda_{\max}(D_L) > 0$ .

*Theorem 1:* Consider the mechanical system described by (1) and (2), using the control law (7), the following holds for any  $(q(0), \dot{q}(0)) \in \Omega_h$ :

- i)  $s$  converges to a set containing the origin with the convergence rate at least as fast as  $e^{-\nu t}$ ;
- ii)  $e_q$  and  $\dot{e}_q$  asymptotically converge to 0 as  $t \rightarrow \infty$ , and
- iii)  $e_\lambda$  and  $\tau$  are bounded for all  $t \geq 0$ .

*Proof:*

- i) The closed-loop system dynamics can be rewritten as

$$D L \dot{s} = B\tau - (D L \ddot{q}_r^1 + C_1 \dot{q}_r^1 + G + d) - C_1 s + J^T \lambda \quad (8)$$

where  $C_1 = D(q^1)\dot{L}(q^1) + C(q^1, \dot{q}^1)L(q^1)$ .

Substituting (7) into (8), the closed-loop dynamic equation is obtained

$$D_L \dot{s} = -L^T K L s - \frac{s \Phi^2}{\|s\| \Phi + \delta} - \xi - C_L s \quad (9)$$

where  $\xi = D_L \ddot{q}_r^1 + C_L \dot{q}_r^1 + G_L + d_L$ ,  $C_L = L^T C_1$ ,  $G_L = L^T G$ , and  $d_L = L^T d$ .

Consider the Lyapunov candidate function

$$V = \frac{1}{2} s^T D_L s. \quad (10)$$

From Property 3, we have  $(1/2)\lambda_{\min}(D_L)s^T s \leq V \leq (1/2)\lambda_{\max}(D_L)s^T s$ . By using Property 4, the time derivative of  $V$  along the trajectory of (9) is

$$\dot{V} = -s^T L^T K L s - s^T \xi - \frac{\|s\|^2 \Phi^2}{\|s\| \Phi + \delta}$$

$$\leq -s^T L^T K L s + \delta.$$

Therefore, we arrive at  $\dot{V} \leq -\nu V + \delta$ . Thus,  $s$  converges to a set containing the origin with a rate at least as fast as  $e^{-\nu t}$ .

ii) Integrating both sides of the above equation gives

$$V(t) - V(0) = - \int_0^t s^T L^T K L s ds + \int_0^t \delta ds \leq \infty. \quad (11)$$

Thus,  $V$  is bounded, which implies that  $s \in L_\infty^{n-m}$ .

From (11), we have  $\int_0^t s^T L^T K L s ds \leq V(0) - V(t) + a$ , which leads to  $s \in L_2^{n-m}$ . From  $s = \dot{e}_z + \rho_1 e_z$ , it can be obtained that  $e_{q1}, \dot{e}_{q1} \in L_\infty^{n-m}$ .

As we have established  $e_{q1}, \dot{e}_{q1} \in L_\infty$ , from Assumption 1, we conclude that  $q^1(t), \dot{q}^1(t), \ddot{q}^1(t), \ddot{q}_r^1(t) \in L_\infty^{n-m}$ , and  $\dot{q} \in L_\infty$ .

Therefore, all the signals on the right-hand side of (9) are bounded and we can conclude that  $\dot{s}$  and therefore  $\ddot{q}^1$  are bounded. Thus,  $s \rightarrow 0$  as  $t \rightarrow \infty$  can be obtained. Consequently, we have  $e_{q1} \rightarrow 0, \dot{e}_{q1} \rightarrow 0$  as  $t \rightarrow \infty$ . It follows that  $e_q, \dot{e}_q \rightarrow 0$  as  $t \rightarrow \infty$ .

iii) Substituting the control (7) into the reduced order dynamic system model (3) yields

$$\begin{aligned} J^T \left[ (K_\lambda + 1)e_\lambda + K_I \int e_\lambda dt \right] \\ = DL\ddot{q}^1 + C_1\dot{q}^1 + G + d + KL's + \frac{L^+ s \Phi^2}{\|s\| \Phi + \delta}. \end{aligned} \quad (12)$$

Since  $s, q^1, \dot{q}^1, \ddot{q}^1$ , and  $d$  are all bounded, the right-hand side of (12) is bounded, i.e.,  $J^T[(K_\lambda + 1)e_\lambda + K_I \int e_\lambda dt] = \alpha(q, \dot{q}, \ddot{q}, \ddot{q}_r, \ddot{q}_r^1), \alpha \in L_\infty$ .

Let  $\int e_\lambda dt = E_\lambda$ , then  $\dot{E}_\lambda = e_\lambda$ . By appropriate choosing  $K_\lambda = \text{diag}[k_{\lambda,i}], k_{\lambda,i} > -1$  and  $K_I = \text{diag}[k_{I,i}], k_{I,i} > 0$  to make  $E_i(p) = (1/(k_{\lambda,i} + 1)p + k_{I,i}), p = d/dt$  a strictly proper exponential stable transfer function, it can be concluded that  $\int e_\lambda dt \in L_\infty, e_\lambda \in L_\infty$ , and the size of  $e_\lambda$  can be adjusted by choosing the proper gain matrices  $K_\lambda$  and  $K_I$ .

Since  $s, q^1, \dot{q}^1, \ddot{q}^1, \ddot{q}_r^1, \ddot{q}_r^1, e_\lambda$ , and  $\int e_\lambda dt$  are all bounded, it is easy to conclude that  $\tau$  is bounded from (7).  $\square$

## B. Robust Control of Nonholonomic Mechanical Systems

*Assumption 2:* The matrix  $H^T(q)B(q)$  is of full rank, which guarantees all  $n - l$  degrees of freedom can be (independently) actuated.

This assumption holds for a large class of nonholonomic mechanical systems such as nonholonomic Caplygin systems, which include a vertical wheel rolling without slipping on a plane surface, a mobile wheeled robot moving on a horizontal plane, and a knife edge moving in point contact on a plane surface, etc. In these systems, the internal state  $\dot{z}(q)$  and variable  $z(q)$  possess practical physical meanings.

It has been proven that the nonholonomic system (1) and (4) cannot be stabilized to a single point using smooth state feedback [20]. It can only be stabilized to a manifold of dimension  $l$  due to the existence of  $l$  nonholonomic constraints. Though the nonsmooth feedback laws [21] or time-varying feedback laws [22] can be used to stabilize these systems to a point, it is fair to say that these approaches are not exclusive. It is worth mentioning that different control objectives may also be pursued, such as stabilization to manifolds of equilibrium points (as opposed to a single equilibrium position) or to trajectories.

By appropriate selecting a set of  $(n - l)$  vector of variables  $z(q)$  and  $\dot{z}(q)$ , the control objective can be specified as: given a desired  $z_d, \dot{z}_d$ , determine a control law such that for any  $(q(0), \dot{q}(0)) \in \Omega_{nh}, \lambda$

remains bounded and  $z(q), \dot{q}$  asymptotically converge to a manifold  $\Omega_{nhd}$  specified as

$$\Omega_{nhd} = \{(q, \dot{q}) | z(q) = z_d, \dot{q} = H(q)\dot{z}_d\}. \quad (13)$$

The variable  $z(q)$  can be thought of as  $(n - l)$  "output equations" of the nonholonomic system. For simplicity of notations and explanations, some representations of the variables same as those in the study of holonomic control designs in Section III-A will be employed in this section also.

*Assumption 3:* The desired reference trajectory  $z_d(t)$  is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order.

Again, define the follow auxiliary variables  $e_z = z - z_d, \dot{z}_r = \dot{z} - \rho_1 e_z, s = \dot{e}_z + \rho_1 e_z$ .

Consider the control law given by

$$\tau = (H^T B)^{-1} \left[ -H^T K H s - \frac{s \Psi^2}{\|s\| \Psi + \delta} \right] \quad (14)$$

where  $K$  is positive definite, and  $\Psi = \|H(q)\| \{c_1 \|(d/dt)[H(q)\dot{z}_r]\| + (c_2 + c_3 \|\dot{q}\|) \|H(q)\dot{z}_r\| + c_4 + c_5\}, \delta(t) > 0$  such that  $\int_0^t \delta(\omega) d\omega = a < \infty$ .

Since both dynamic equation (3) as well as (5) strongly resemble and possess similar properties as shown in Properties 3–5, the same procedures of analysis as in Theorem 1 can be employed and the following result can be obtained.

Define  $\nu = \lambda_{\min}(H^T K H) / \lambda_{\max}(D_H) > 0$ .

*Theorem 2:* Consider the mechanical system described by (1) and (4), using the control law (14), the following holds for any  $(q(0), \dot{q}(0)) \in \Omega_{nh}$ :

- i)  $s$  converges to a set containing the origin with the convergence rate at least as fast as  $e^{-\nu t}$ ;
- ii)  $e_z$  and  $\dot{e}_z$  asymptotically converge to 0 as  $t \rightarrow \infty$ ;
- iii)  $\lambda$  and  $\tau$  are bounded for all  $t \geq 0$ .

*Proof:* The proof is similar to that of Theorem 1.  $\square$

*Remark 1:* In Theorem 1 or 2, by choosing  $\delta$  as a time-varying positive scalar with its time integral bounded, asymptotically stability can be obtained. If  $\delta$  is chosen to converge exponentially to zero,  $s$  and  $e_q, \dot{e}_q$  or  $z, \dot{z}$  exponentially converge to zero, and exponential stability is obtained. The control law in these cases are continuous for any finite time  $t$ , the control laws tend to the ideal control laws as  $t \rightarrow \infty$ . Therefore, control chattering will appear when  $t \rightarrow \infty$ . To remove control chattering, nonzero  $\delta$  must be used. However, then only <AU: PLEASE SPELL OUT..?> (GUUB) stability can be guaranteed and asymptotic stability is lost.

## IV. ADAPTIVE ROBUST CONTROL DESIGN

In developing control laws (7) and (14),  $c_i, 1 \leq i \leq 5$  are supposed to be known. However, in reality, these constants cannot be obtained easily. Although any fixed large  $c_i$  can guarantee good performance, it is not recommended in practice as large  $c_i$  imply, in general, high noise amplification and high cost of control. Therefore, it is necessary to develop a control law which does not require the knowledge of  $c_i, 1 \leq i \leq 5$ .

### A. ARC of Holonomic Mechanical Systems

Consider the adaptive robust control law given by

$$\begin{aligned} B\tau = -KLs - \sum_{i=1}^5 \frac{L^+ s \hat{c}_i \Phi_i^2}{\|s\| \Phi_i + \delta_i} - J^T \lambda_c \quad (15) \\ \dot{\hat{c}}_i = -\sigma_i \hat{c}_i + \frac{\gamma_i \Phi_i^2 \|s\|^2}{\Phi_i \|s\| + \delta_i}, \quad i = 1, \dots, 5 \quad (16) \end{aligned}$$

where  $\Phi_1 = \|L(q^1)\| \cdot \|(d/dt)[L(q^1)\dot{q}_r^1]\|$ ,  $\Phi_2 = \|L(q^1)\| \cdot \|L(q^1)\dot{q}_r^1\|$ ,  $\Phi_3 = \|L(q^1)\| \cdot \|\dot{q}\| \cdot \|L(q^1)\dot{q}_r^1\|$ ,  $\Phi_4 = \Phi_5 = \|L(q^1)\|$ ,  $\gamma_i > 0$ ,  $\delta_i(t) > 0$ , and  $\sigma_i(t) > 0$  such that  $\int_0^t \delta_i(\omega)d\omega = a_i < \infty$  and  $\int_0^t \sigma_i(\omega)d\omega = b_i < \infty$ .

Define  $\nu = \min\{\lambda_{\min}(L^T K L), (\sigma_i/2\gamma_i)\} / \max\{\lambda_{\max}(D_L), (1/\gamma_i)\} > 0$ .

**Theorem 3:** Consider the mechanical system described by (1) and (2), using the control law (15) and adaptation law (16), the following holds for any  $(q(0), \dot{q}(0)) \in \Omega_h$ :

- i)  $s$  converges to a set containing the origin with the convergence rate at least as fast as  $e^{-\nu t}$ ;
- ii)  $e_q$  and  $\dot{e}_q$  asymptotically converge to 0 as  $t \rightarrow \infty$ ;
- iii)  $e_\lambda$  and  $\tau$  are bounded for all  $t \geq 0$ .

*Proof:* (i). Substituting (15) into (8), the closed-loop dynamic equation is obtained

$$D_L \dot{s} = -L^T K L s - \sum_{i=1}^5 \frac{s \hat{c}_i \Phi_i^2}{\|s\| \Phi_i + \delta_i} - \xi - C_L s. \quad (17)$$

Consider the Lyapunov candidate function

$$V = \frac{1}{2} s^T D_L s + \sum_{i=1}^5 \frac{1}{2\gamma_i} \hat{c}_i^2. \quad (18)$$

Its time derivative along the trajectory of (17) is

$$\dot{V} \leq -s^T L^T K L s - \sum_{i=1}^5 \frac{\sigma_i}{2\gamma_i} \hat{c}_i^2 + \sum_{i=1}^5 \left( c_i \delta_i + \frac{\sigma_i \hat{c}_i^2}{4\gamma_i} \right). \quad (19)$$

Therefore,  $\dot{V} \leq -\nu V + \bar{\delta}$ , which implies that  $s$  converges to a set containing the origin with a rate at least as fast as  $e^{-\nu t}$ .

Following the similar procedures as in the Proof of Theorem 1, (ii) and (iii) can be proved, i.e.,  $e_q$  and  $\dot{e}_q \rightarrow 0$  as  $t \rightarrow \infty$ ,  $e_\lambda$ , and  $\tau$  are uniformly ultimately bounded for all time  $t \geq 0$ .  $\square$

### B. ARC for Nonholonomic Mechanical Systems

Consider the adaptive robust control law given by

$$\tau = (H^T B)^{-1} \left( -H^T K H s - \sum_{i=1}^7 \frac{s \hat{c}_i \Psi_i^2}{\|s\| \Psi_i + \delta_i} \right) \quad (20)$$

$$\dot{\hat{c}}_i = -\sigma_i \hat{c}_i + \frac{\gamma_i \Psi_i^2 \|s\|^2}{\Psi_i \|s\| + \delta_i}, \quad i = 1, \dots, 7 \quad (21)$$

where  $\Psi_1 = \|H(q)\| \cdot \|(d/dt)[H(q)\dot{z}_r]\|$ ,  $\Psi_2 = \|H(q)\| \cdot \|H(q)\dot{z}_r\|$ ,  $\Psi_3 = \|H(q)\| \cdot \|\dot{q}\| \cdot \|H(q)\dot{z}_r\|$ ,  $\Psi_4 = \Psi_5 = \|H(q)\|$ ,  $\gamma_i > 0$ ,  $\delta_i(t) > 0$ , and  $\sigma_i(t) > 0$  such that  $\int_0^t \delta_i(\omega)d\omega = a_i < \infty$  and  $\int_0^t \sigma_i(\omega)d\omega = b_i < \infty$ .

The same procedures of analysis as in Theorem 3 can be employed and the following result can be obtained. Define  $\nu = \min\{\lambda_{\min}(H^T K H), (\sigma_i/2\gamma_i)\} / \max\{\lambda_{\max}(D_H), (1/\gamma_i)\} > 0$ .

**Theorem 4:** Consider the mechanical system described by (1) and (4), using the control law (20) and adaptation law (21), then the following holds for any  $(q(0), \dot{q}(0)) \in \Omega_{nh}$ :

- i)  $s$  converges to a set containing the origin with the convergence rate at least as fast as  $e^{-\nu t}$ ;
- ii)  $e_z$  and  $\dot{e}_z$  asymptotically converge to 0 as  $t \rightarrow \infty$ ;
- iii)  $\lambda$  and  $\tau$  are bounded for all  $t \geq 0$ .

*Proof:* The proof is similar to that of Theorem 3.  $\square$

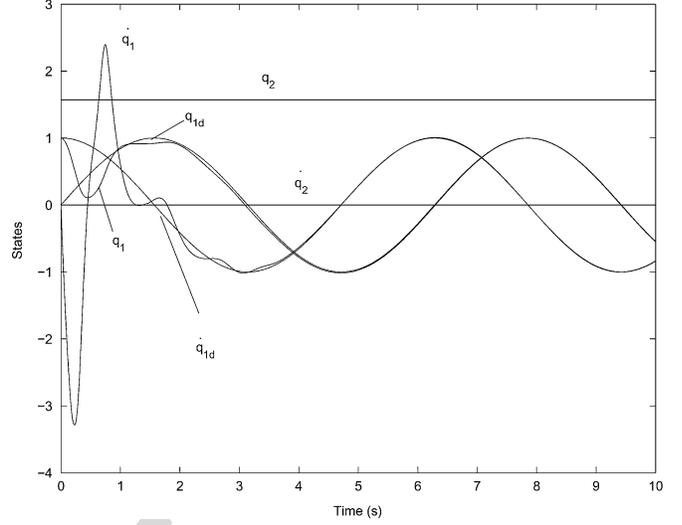


Fig. 1. Simulated holonomic system's responses.

## V. SIMULATION RESULTS

### A. Control of a Holonomic Constrained Robot

Simulations are carried out on a two-link robotic manipulator with a circular path constraint, whose dynamics is given by

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C_{12}\dot{q}_2 & -C_{12}(\dot{q}_1 + \dot{q}_2) \\ C_{12}\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

where  $D_{11} = (m_1 + m_2)l_1^2 + m_2l_2^2 + 2m_2l_1l_2 \cos q_2$ ,  $D_{12} = m_2l_1l_2 \cos q_2$ ,  $D_{22} = m_2l_2^2$ ,  $C_{12} = m_2l_1l_2 \sin q_2$ ,  $G_1 = (m_1 + m_2)l_1g \cos q_1 + m_2l_2g \cos(q_1 + q_2)$ , and  $G_2 = m_2l_2g \cos(q_1 + q_2)$ .

The constraint is a circle in the work space whose center coincides with axis of rotation of the first link and is expressed as  $h(q) = l_1^2 + l_2^2 + 2l_1l_2 \cos q_2 - r^2 = 0$ . Thus,  $q_2 = \cos^{-1}[(r^2 - (l_1^2 + l_2^2))/(2l_1l_2)] = q_2^*$  is a constant. The Jacobian matrix is  $J(q) = [0 \quad -2l_1l_2 \sin q_2]$ ,  $L(q^1) = [1 \quad 0]^T$ , and  $q^1 = q_1$ ,  $q^2 = q_2$ . For the convenience of simulation, the nominal parameters of the robot system are taken as  $m_1 = 1$  Kg,  $m_2 = 2$  Kg,  $l_1 = l_2 = 1$  m,  $r = \sqrt{2}$  m, and  $g = 9.8$  m/s<sup>2</sup>, the initial conditions are taken as  $q(0) = [1.0, 0]^T$ ,  $\dot{q}(0) = [0, 0]^T$ , and the desired manifold  $\Omega_{hd}$  is chosen as  $\Omega_{hd} = \{(q, \dot{q}, \lambda) | q^1 = \sin(t), q^2 = \cos(t), \lambda = 10\}$ .

Using the controller (15) and adaptation law (16), the control gain  $K$  is selected as  $K = \text{diag}[1, 1]$ , and the force control gain  $K_\lambda$  and  $K_I$  are selected as  $K_\lambda = 0.1$ ,  $K_I = 1$ , and  $\rho_1$  is chosen as  $\rho_1 = 5$ . The adaptation gain in adaptation law (16) is chosen as  $\gamma_i = 0.1$  and  $\sigma_i = \delta_i = (1/(1+t)^2)$ . The results of the simulation are shown in Figs. 1–3. Fig. 1 shows the system responses of the simulated holonomic constrained robot. Fig. 2 shows the force tacking error. It can be seen that the motion error converges to zero as desired and the force error remains bounded simultaneously. The torques exerted at the constrained robot are given by Fig. 3. It can be seen that all signals in closed-loop are bounded.

### B. Control of a Nonholonomic Constrained Robot

A simplified model of a mobile wheeled robot moving on a horizontal plane, constituted by a rigid trolley equipped with nondeformable wheel, as given in details in [3], is used to verify the validity

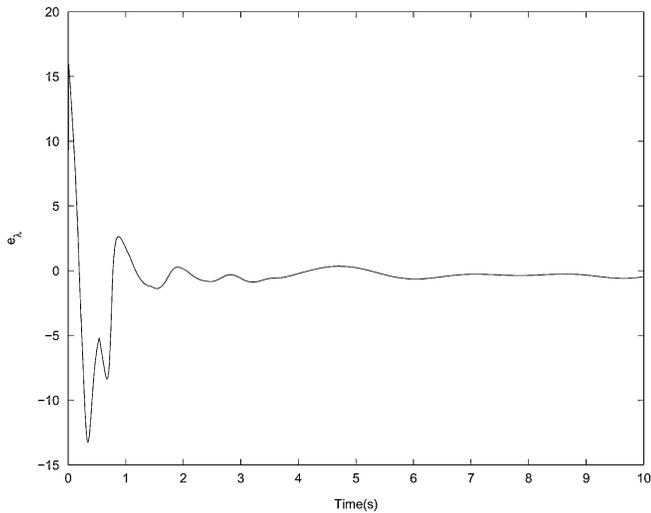
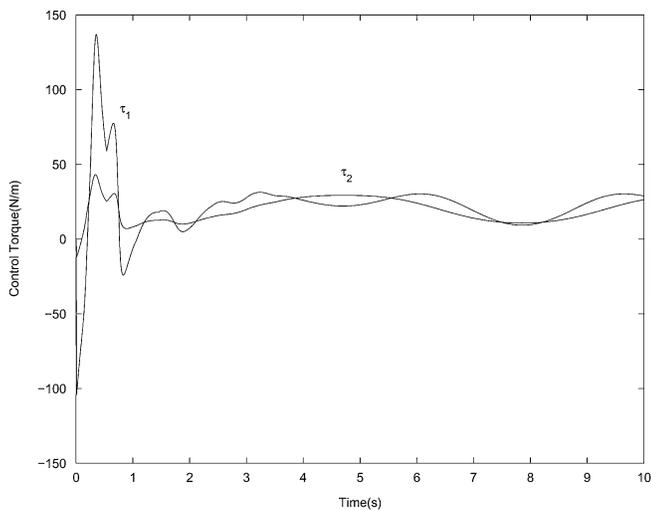

 Fig. 2.  $e_\lambda$  of the simulated holonomic system.


Fig. 3. Control torques of the simulated holonomic system.

of the proposed controllers. The dynamic model can be expressed as [3]

$$\begin{aligned} m\ddot{x} &= \lambda \cos \theta - \frac{1}{P}(\tau_1 + \tau_2) \sin \theta \\ m\ddot{y} &= \lambda \sin \theta + \frac{1}{P}(\tau_1 + \tau_2) \cos \theta \\ I\ddot{\theta} &= \frac{L}{P}(\tau_1 - \tau_2) \end{aligned}$$

where  $x, y$  are coordinates in an inertial frame,  $\theta$  as an orientation of the wheel with respect to the inertial frame,  $m$  is the mass of the robot, and  $I$  is its inertial moment around the vertical axis,  $P$  is the radius of the wheels and  $2L$  the length of the front wheels, and  $\tau_i, i = 1, 2$  is the torques provided by the motors. For simplicity, we set  $P = L = 1$ .

The nonholonomic constraint is written as  $\dot{x} \cos \theta + \dot{y} \sin \theta = 0$ . The matrix  $J(q)$  is  $J(q) = [\cos \theta \ \sin \theta \ 0]$ ,  $q = [x \ y \ \theta]^T$ . The “outputs” are chosen as  $z(q) = [y \ \theta]^T$ . The matrix  $H(q)$  is chosen as  $H = [H_1 \ H_2]$ ,  $H_1 = [-\tan \theta \ 1 \ 0]^T$  and  $H_2 = [0 \ 0 \ 1]^T$ . The relation  $\dot{q} = H(q)\dot{z}$  is satisfied. The desired manifold  $\Omega_{nhd}$  is chosen as  $\Omega_{nhd} = \{(q, \dot{q}, \lambda) \mid z(q) = \sin(t), \dot{q} = H(q) \cos(t)\}$ .

In order to simulate, it can be easily derived  $D(q) = \text{diag}[m, m, I]$ ,  $C(q, \dot{q}) = 0$ , and  $G(q) = 0$ . In the simulation, we

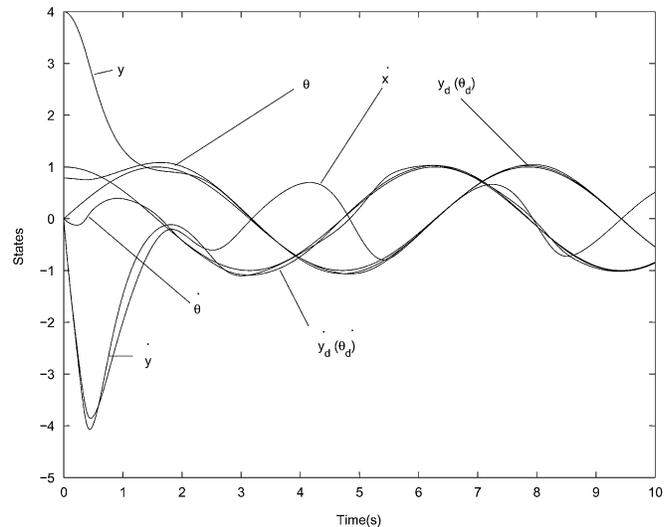


Fig. 4. Simulated nonholonomic system's responses.

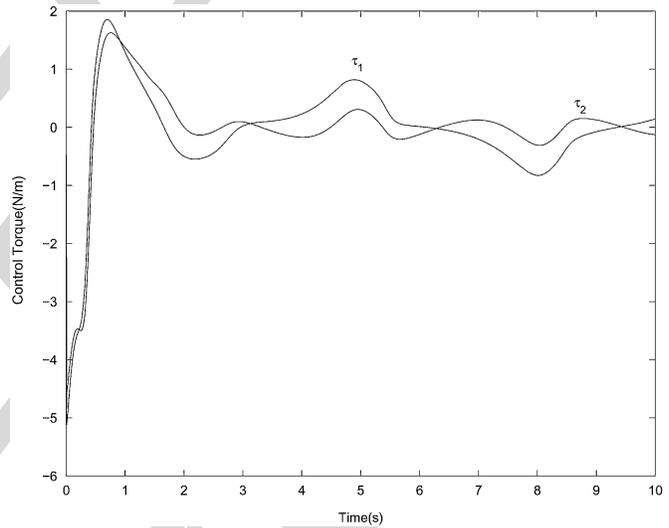


Fig. 5. Control torques of the simulated nonholonomic system.

assume the real parameter  $m = 0.5$ ,  $I = 0.5$ ,  $q(0) = [0, 4, 0.785]^T$ ,  $\dot{q}(0) = [0, 0, 0]^T$ , and  $\rho_1 = \text{diag}[2, 2]$ . By Theorem 4, the control gain  $K$  is selected as  $K = \text{diag}[1, 1]$ . The adaptation gain in adaptation law (21) is chosen as  $\gamma_i = 0.1$  and  $\sigma_i = \delta_i = (1/(1+t))^2$ .

Using the controller (20) and adaptation law (21), the results of the simulation are shown in Figs. 4–5. Fig. 4 shows the responses of the simulated nonholonomic constrained robot. It can be seen that the system states converge to the desired manifold  $\Omega_{nhd}$ . And the torques exerted at the mobile robot are given by Fig. 5. It can be seen that the control inputs are bounded.

In the simulations, the parameters are all selected by the authors at will to demonstrate the performance of the proposed method. Different control performance can be achieved by adjusting parameter adaptation gains and other factors by trial and error. The best set of parameters, however, depends very much on individual applications that could differ significantly from the examples here. For this reason, the simulations are conducted with only one set of controller parameters. It can be seen that the control performance is satisfactory for both holonomic and nonholonomic constrained systems.

## VI. CONCLUSION

In this short paper, simple and effective robust control strategies have been presented systematically to control the holonomic mechanical systems and a class of nonholonomic systems in the presence of uncertainties and disturbances. All control strategies have been designed to drive the system motion converge to the desired manifold and at the same time guarantee the boundedness of the constrained force. The proposed controllers are nonregressor based and require no information on the system dynamics. PI type feedback has been used in the force control for the benefit of the real time implementation. Simulation results have shown the effectiveness of the proposed controller.

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