

Oppenheim Lecture 2017 and Workshop on Ergodic Theory & Dynamical Systems

(held in conjunction with the Oppenheim Lecture)

Date: 14 to 16 February 2017

Venue: Department of Mathematics
Block S17, Seminar Room 1 (#04-06)
National University of Singapore

Jointly
organized by



Department of Mathematics
Faculty of Science



Programme

Tuesday, 14 February 2017

- 9.30am – 10.30am **Quantitative Oppenheim conjecture for S-arithmetic case**
Seonhee Lim, Seoul National University
- 10.30am – 11:00am **Refreshments @ Staff Lounge**
- 11:00am – 12:00pm **Effective versions of Oppenheim's conjecture for generic forms**
Anish Ghosh, Tata Institute of Fundamental Research
- 2.30pm – 3.30pm **Several special cases of Möbius disjointness**
Zhiren Wang, Pennsylvania State University
- 3.30pm – 4.00pm **Refreshments @ Staff Lounge**
- 4.00pm – 5.00pm **Higher rank diagonalizable flows**
Elon Lindenstrauss, The Hebrew University of Jerusalem

Wednesday, 15 February 2017

- 9.30am – 10.30am **Dynamics of complex Henon maps in higher dimension**
Dinh Tien Cuong, National University of Singapore
- 10.30am – 11:00am **Refreshments @ Staff Lounge**
- 11:00am – 12:00pm **Hausdorff dimension of the graphs of Weierstrass functions**
Weixiao Shen, Fudan University
- 12.00pm **Lunch reception @ Staff Lounge**
- 2.00pm – 3.30pm **Oppenheim Lecture / Q & A**
On an effective proof of the Oppenheim Conjecture (joint work with G. A. Margulis)
Elon Lindenstrauss, The Hebrew University of Jerusalem
- 3.30pm – 4.30pm **Refreshments @ Staff Lounge**
- 6.00pm **Workshop dinner (for speakers and invited guests)**

Thursday, 16 February 2017

- 9.30am – 10.30am **Bounded orbits of diagonalizable flows on homogeneous spaces**
Jinpeng An, Peking University
- 10.30am – 11.30am **Endomorphisms of complex varieties - a minimal model program approach**
De-Qi Zhang, National University of Singapore
- End of Workshop-----
- 3.00pm **Conversation with Prof Elon Lindenstrauss**

Abstracts

Oppenheim Lecture, 15 February, 2.00pm

On an effective proof of the Oppenheim Conjecture (joint work with G. A. Margulis)
Elon Lindenstrauss, The Hebrew University of Jerusalem

Margulis' proof in mid 80's of the longstanding Oppenheim Conjecture concerning values of indefinite quadratic forms at integer points using homogeneous dynamics, and the subsequent strengthening of this result by Dani and Margulis, were an important milestone in the development of the subject. I will describe joint work with Margulis which gives an effective proof of the Oppenheim conjecture, and explain how it relates to the quantitative study of unipotent flows.

Workshop on Ergodic Theory & Dynamical Systems, 14 to 16 February

Tuesday 14 February, 9.30am

Quantitative Oppenheim conjecture for S-arithmetic case
Seonhee Lim, Seoul National University

In this talk, we will explain how we generalize a theorem of Eskin-Margulis-Mozes to an S-arithmetic setup. Suppose that we are given a finite set of places over \mathbb{Q} containing the archimedean place. For a given irrational isotropic form q of rank at least 4 and not of signature $(2,2)$, we prove an asymptotic formula on the number of S-integral vectors in a ball of radius $T=(T_p)$ as each T_p tends to infinity. We will also explain the difficulties which arise in signature $(2,2)$ case. This is a joint work with Keivan Mallahi-Karai and Jiyoung Han.

Tuesday 14 February, 11.00am

Effective versions of Oppenheim's conjecture for generic forms.
Anish Ghosh, Tata Institute of Fundamental Research

I will discuss a method of obtaining effective results for Oppenheim's conjecture and related problems for generic quadratic forms using ergodic theorems for semisimple groups and a duality phenomena. Joint work with Alexander Gorodnik and Amos Nevo.

Tuesday 14 February, 2.30pm

Several special cases of Möbius disjointness
Zhiren Wang, Pennsylvania State University

Sarnak's Möbius disjointness conjecture speculates that the Möbius sequence is disjoint to all topological dynamical systems of zero topological entropy. In this talk we will discuss several special cases of this conjecture obtained by applying a number-theoretical theorem of Matomäki-Radziwiłł-Tao to dynamical settings. Part of the talk is based on joint work with Wen Huang and Guohua Zhang.

Tuesday 14 February, 4.00pm

Higher rank diagonalizable flows
Elon Lindenstrauss, The Hebrew University of Jerusalem

The proof of the Oppenheim conjecture hinges on the rigidity properties of unipotent flows, namely the scarcity of closed invariant sets and invariant measures for such actions. In contrast, one-parameter diagonalizable groups have an abundance of invariant sets and measures.

Furstenberg on the one hand and Cassel and Swinnerton-Dyers on the other understood from two different perspectives over 50 years ago that the situation changes drastically if one considers action of diagonalizable groups of two or more parameters. While we have at the moment only a partial understanding of the rigidity properties of such actions, even the partial results we know have strong applications towards arithmetic quantum unique ergodicity, distribution of integer points in varieties, and diophantine questions.

Wednesday 15 February, 9.30am

Dynamics of complex Henon maps in higher dimension
Dinh Tien Cuong, National University of Singapore

I will discuss dynamical properties of complex Henon maps in higher dimension, including the unique ergodicity of the Julia set and the equidistribution of periodic points. The main difficulties are (1) the system is not uniformly hyperbolic; (2) we need to work with sub-varieties of arbitrary dimension, codimension, degree, and their intersections. I will introduce the main tools and discuss other applications. This talk is based on my works with Nessim Sibony, Viet-Anh Nguyen and Truong Trung Tuyen.

Wednesday 15 February, 11.00am

Hausdorff dimension of the graphs of Weierstrass functions
Weixiao Shen, Fudan University

We will calculate the Hausdorff dimension of the classical Weierstrass function $W(x) = \sum \lambda^n \cos(b^n x)$ for $b \geq 2$ integer and all $\lambda \in (1/b, 1)$. The recent work of Baranski-Barany-Romanowska obtained the conjectured value when λ is sufficiently close to one, applying earlier results due to Ledrappier and Tsujii respectively. We will show how to modify the argument of Tsujii to cover all parameters λ .

Thursday 16 February, 9.30am

Bounded orbits of diagonalizable flows on homogeneous spaces
Jinpeng An, Peking University

Let G be a Lie group with a non-compact lattice Γ and a diagonalizable one-parameter subgroup F . A theorem of Kleinbock and Margulis states that the points in G/Γ with bounded F -orbits form a set of full Hausdorff dimension. For countably many diagonalizable one-parameter subgroups $\{F_k\}$ of G , is it true that the points in G/Γ whose F_k -orbits are all bounded still form a set of full Hausdorff dimension? We expect an affirmative answer to this question. In this talk, I will report some recent progress and its relations to Diophantine approximation.

Thursday 16 February, 10.30am

Endomorphisms of complex varieties - a minimal model program approach
De-Qi Zhang, National University of Singapore

We show the existence of equivariant minimal model program for polarised endomorphisms or solvable automorphism groups of "maximal rank". Applications include the characterizations of abelian varieties or their quotients.