IMPRINTS

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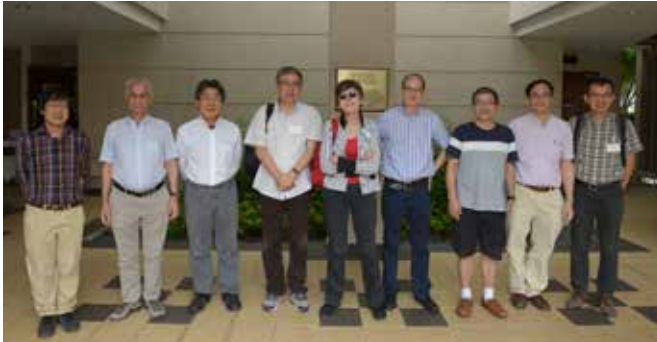**NUS** National University of Singapore

Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions >>>

From left: Keiji Oguiso, Nessim Sibony, Yujiro Kawamata, Gang TIAN, Hélène Esnault, De-Qi ZHANG, Shou-Wu ZHANG, Wing Keung TO and Tien Cuong Dinh

[Editor's note: From 3 to 28 January 2017, the Institute hosted the program "Higher Dimensional Algebraic Geometry, Holomorphic Dynamics and Their Interactions". The program organizers and some of the program visitors contributed this invited article to Imprints.*]*

From an algebro-geometric point of view, one likes to classify compact varieties according to their isomorphism classes or birational classes. The former classes are more rigid and the latter ones are more flexible in the following sense. If X is a variety, one can blow up a subvariety of X and get a new variety X′. Of course, X and X′ are not isomorphic to each other, but their relation is very clear -- they are the same modulo some proper nowhere dense subsets. Therefore, it makes more sense to determine the birational classes instead of isomorphism classes of varieties. In recent years there have been breakthroughs in the classification

theory of higher dimensional compact algebraic varieties and complex manifolds. These results in algebraic geometry have profound influence on other areas of mathematics, including the study of higher dimensional dynamics and number theoretical dynamics.

The typical ways of classifying varieties are to look at the following three canonical fibrations on a variety X. The first fibration is the Iitaka (or Iitaka-Kodaira) fibration I : X → I(X), where its very general fibres F are varieties of Kodaira dimension zero and the base variety I(X) has dimension equal to the Kodaira dimension of X. This fibration reduces the classification of varieties of positive Kodaira dimension to those of Kodaira dimension zero or –∞. The second fibration is the Albanese fibration albX : X → Alb(X), where the codomain Alb(X) is a complex torus of dimension equal to the irregularity of X and hence has vanishing Kodaira dimension and Chern classes. Then to some extent, the classification of varieties is reduced to those of vanishing irregularity. The third fibration is the maximal rationally connected (MRC) fibration r : X → r(X) where the general fibres F are rationally connected varieties and the codomain is a non-uniruled variety. Recall that a variety F is rationally connected if every two general points of F are connected by a rational curve of F; we also recall that rational curves are the simplest curves, being of genus zero. From these three fibrations, the classification of varieties is reduced, to some extent, to the three types of building blocks: varieties of general type, varieties of vanishing Kodaira dimension and irregularity, and rationally connected varieties respectively.

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Next we turn our attention to the study of holomorphic dynamics, where we are interested in the symmetries f on a variety X. It is natural that one tries to apply the above algebro-geometric machineries to the study of automorphisms or endomorphisms f of X, or even birational or rational self-maps f of X. Such an approach provides an attractive immediate program to work on. At the same time, one will encounter some challenging obstacles in this dynamical setting, which still remain to be overcome. The IMS program provided a platform for discussion of some of these and other issues, and it assembled a group of algebraic geometers, complex geometers, arithmetic geometers and holomorphic dynamists. The main activity of the program was a 2-week workshop covering the major areas of algebraic geometry, arithmetic geometry and holomorphic dynamics with algebraic or number theoretic flavors. The program was highlighted by two lecture series by the following mathematicians:

1) Gang Tian of Peking University and Princeton University delivered two lectures elaborating the proof of Yau-Tian-Donaldson conjecture: the existence of a (geometric) Kähler-Einstein metric on a Fano manifold is equivalent to the (algebraic) K-stability condition. He also surveyed some of his related works and some open questions.

2) Shou-Wu Zhang of Princeton University gave two lectures and one colloquium talk covering the state-of-the-art development on several fundamental conjectures in arithmetic geometry and dynamics: the Manin-Mumford conjecture and its dynamical analogue, the Andre-Oort conjecture and Colmez' conjecture, and the ABC conjecture and BSD conjecture. He also illustrated the proof of Junyi Xie of Rennes University (also a program participant) on the Dynamic Manin-Mumford conjecture (proposed by Shou-Wu Zhang) when the endomorphism is a Frobenius lifting, using Peter Scholze's perfectoid space theory.

In addition to the numerous lectures delivered during the workshop fortnight, the program participants had plenty of research interactions and discussions throughout the entire duration of the program, and it is expected that these will lead to some fruitful research collaborations in the near future.

The Institute provided a comfortable and relaxed environment conducive to research, which benefitted the program tremendously. The superb service and untiring attention offered by the Institute's staff are highly appreciated by the program participants. The program organizers would like to take this opportunity to express their thanks to the Institute and its staff members for help in making the program a success.

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